Higher order

(1) Solve $y'' + 5y' + 6y = 0$

Ans

$$y'' + 5y' + 6y = 0$$

$$D^2y + 5Dy + 6y = 0$$

$$(D^2 + 5D + 6)y = 0$$

The auxillary eqn is $m^2 + 5m + 6 = 0$

$$m^2 + 5m + 6 = 0 \qquad 3 \cdot 2 = 6$$

$$(m+3)(m+2) = 0 \qquad 3 + 2 = 5$$

$$m = -3, -2$$

Roots are real & distinct above case:-1

So

$$y_c = c_1 e^{-3x} + c_2 e^{-2x}$$

here $R(x) = 0$ so $y_p = 0$

$$y = y_c + y_p$$

$$y = y_c$$

$$y = c_1 e^{-3x} + c_2 e^{-2x}$$

② solve $y''' + y = 0$

Ans

$$y''' + y = 0 \text{ or}$$

$$\frac{d^3y}{dx^3} + y = 0 \text{ or}$$

$$D^3y + y = 0$$

$$y''' + y = 0$$

$$D^3y + y = 0$$

$$(D^3 + 1)y = 0$$

The auxillary eqn is $m^3 + 1 = 0$

$$m^3 + 1 = 0$$

$$(m+1)(m^2 - m + 1) = 0$$

$$m^2 - m + 1 = 0$$

$$a = 1,\ b = -1,\ c = 1$$

$$\Delta = b^2 - 4ac = 1 - 4(1)(1)$$

$$= -3$$

$$\frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$\frac{1 \pm i\sqrt{3}}{2} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$ comlairz

$a \pm ib$ so $a = \frac{1}{2}$ & $b = \frac{\sqrt{3}}{2}$

$(m+1) = 0 \Rightarrow m = -1$

$-1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y_c = c_1 e^{-x} + e^{ax}\left[c_2 \cos bx + c_3 \sin bx\right]$

$= c_1 e^{-x} + e^{\frac{1}{2}x}\left[c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right]$

Here $R(x) = 0$ so $y_p = 0$

$y = y_c + y_p$

$= c_1 e^{-x} + e^{\frac{1}{2}x}\left[c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right]$

3) Solve $y'' + 4y' + 4y = 0$

Ans $y'' + 4y' + 4y = 0$

$D^2y + 4Dy + 4y = 0$

$(D^2 + 4D + 4)y = 0$

The auxiliary eqn is $m^2 + 4m + 4 = 0$

$$m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0$$

$$m = -2, -2$$

Roots are real & Equal (case :- 2)

$$y_c = (c_1 + c_2 x) e^{-2x}$$

Here $y_p = 0$ because $f(x) = 0$

$$y = y_c + y_p.$$

$$y = (c_1 + c_2 x) e^{-2x}$$

(4)

Solve $y''' + 4y'' - 5y' = 0$ or

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - 5\frac{dy}{dx} = 0$$ or

$$D^3y + 4D^2y - 5Dy = 0$$

Ans

$$y''' + 4y'' - 5y' = 0$$

$$D^3y + 4D^2y - 5Dy = 0$$

(5)

$$(D^3 + 4D^2 + 5D)\, y = 0$$

The auxillary Eqn is

$$m^3 + 4m^2 + 5m = 0$$

$$m[m^2 + 4m - 5] = 0$$

$$m(m+5)(m-1) = 0$$

$$m = 0, -5, 1$$

Roots are real & distinct

$$y_c = c_1 e^{0x} + c_2 e^{-5x} + c_3 e^{x}$$

$$= c_1 + c_2 e^{-5x} + c_3 e^{x}$$

Here R.H.S = 0 so $y_p = 0$

$$y = y_c + y_p$$

$$\boxed{y = c_1 + c_2 e^{-5x} + c_3 e^{x}}$$

(5) Solve $\frac{d^4y}{dx^4} - 81y = 0$

Ans $\frac{d^4y}{dx^4} - 81y = 0$

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$$D^4y - 81y = 0$$

$$(D^4 - 81)y = 0$$

The auxillary eqn is $m^4 - 81 = 0$

$$m^4 - 81 = 0$$

$$(m^2 - 9)(m^2 + 9) = 0 \qquad (i^2 = -1)$$

$$m^2 - 9 = 0 \qquad m^2 - i^2 9 = 0$$

$$(m-3)(m+3) = 0 \qquad (m - 3i)(m + 3i) = 0$$

$$m = 3, -3, \pm 3i$$

$$0 \pm 3i \Rightarrow a \pm ib$$

$$a = 0, \quad b = 3$$

$$y_c = c_1 e^{3x} + c_2 e^{-3x} + e^{0x}[c_3 \cos 3x + c_4 \sin 3x]$$

$$= c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos 3x + c_4 \sin 3x$$

here Rus $= 0$ so $y_p = 0$

$$y = y_c + y_p$$

$$y = c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos 3x + c_4 \sin 3x$$

(6) Solve $y''' - 6y'' + 6y' - y = 0$

Ans $D^3y - 6D^2y + 6Dy - y = 0$

$(D^3 - 6D^2 + 6D - 1)y = 0$

The auxillary eqn is

$m^3 - 6m^2 + 6m - 1 = 0$

$(m-1)(m^2 - 5m + 1) = 0$

$m = 1$ $\quad \Delta = b^2 - 4ac \quad a = 1,\ b = -5,\ c = 1$

$= 25 - 4(1)(1)$

$= 25 - 4$

$= 21$

$\Delta = 21$

$\dfrac{-b \pm \sqrt{\Delta}}{2a}$

$= \dfrac{-(-5) \pm \sqrt{21}}{2(1)} = \dfrac{5 \pm \sqrt{21}}{2}$

$= \dfrac{5}{2} \pm \dfrac{\sqrt{21}}{2}$

$y_c = c_1 e^{x} + e^{\frac{5}{2}x}\left[c_2 \cosh \dfrac{\sqrt{21}}{2}x + c_3 \sinh \dfrac{\sqrt{21}}{2}x\right]$

(8)

$$y_c = c_1 e^{x} + e^{\frac{5}{2}x}\left[c_2 \cosh \frac{\sqrt{21}}{2} x + c_3 \sinh \frac{\sqrt{21}}{2} x\right]$$

here $f(x) = 0$ so $\underline{y_P = 0}$

$y = y_c + y_P$

$$= c_1 e^{x} + e^{\frac{5}{2}x}\left[c_2 \cosh \frac{\sqrt{21}}{2} x + c_3 \sinh \frac{\sqrt{21}}{2} x\right]$$

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