

Higher order

$$D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}, \quad D^3 = \frac{d^3}{dx^3}, \dots$$

$$Dy = \frac{dy}{dx}, \quad D^2y = \frac{d^2y}{dx^2}$$

$$y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2}$$

Case 1:- $R(x) = 0$ $R(x)$ is Right hand side

$$y = y_c + y_p$$

when $R(x) = 0$ then $y_p = 0$

$$y = y_c$$

Case 2:- when $R(x) \neq 0$ then $y_p \neq 0$ so

$$y = y_c + y_p$$

① If roots are real & distinct then

$$y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots + c_n e^{m_n x}$$

(2)

(2) If two roots are real & equal & $n-2$ roots are distinct then

$$y_c = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x} + \dots + c_n e^{m_n x}$$

(3) If three roots are real & equal

$$y_c = (c_1 + c_2 x + c_3 x^2) e^{m_1 x} + c_4 e^{m_4 x} + \dots + c_n e^{m_n x}$$

(4) If roots are complex then

$$y_c = e^{\gamma x} [c_1 \cos bx + c_2 \sin bx] + c_3 e^{m_3 x} + \dots$$

(5) If two complex conjugate then

$$y_c = e^{\gamma x} [(c_1 + i c_2 x) \cos bx + (c_3 + i c_4 x) \sin bx] + c_4 e^{m_4 x} + c_5 e^{m_5 x} + \dots + c_n e^{m_n x}$$