SHANTILAL SHAH ENGINEERING COLLEGE , BHAVNAGAR

BE Sem-II (All Branches) Mathematics-II(3110015)

Tutorial-3 (Ex-1 to 22) Topic : Vector Calculus

Ex-1 Find the parametric curve of following:

1. $x^2 + y^2 = a^2$ in R^2 ,
2. $y = x^2$ in R^2
3. Line segment from the point (a,b) to (c,d).

Ex-2 Find the length of curve of the circular helix $\bar{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$.

Ex-3 Find the arc length parametrization of the line $x = 3t + 2, y = 2t - 1$ that has reference point $(2,-1)$ and the same orientation as the original line.

Ex-4 Define: Gradient, Curl and divergence.

Ex-5 Find the directional derivative of $f(x,y) = xy + xe^y + \cos(xy)$ at the point $p(1,0)$ in the direction of $\bar{u} = 3\hat{i} - 4\hat{j}$

Ex-6 Determine the directional derivative of $f(x,y,z) = x^2 + y^2 + 2z^2$ at the point $P(1,1,-1)$ in the direction of line from P to $Q(1,2,3)$.

Ex-7 If $\varphi(x,y,z) = xyz - 2y^2z + x^2z^2$, find $div(grad\varphi)$ at the point $P(1,2,3)$.

Ex-8 Find $div(\vec{F})$ and $Curl(\vec{F})$, where $\vec{F} = grad(x^3 + y^3 + z^3 3xyz)$.

Ex-9 Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

Ex-10 Find the value of constant λ suh that $\vec{F} = (2x^2y^2 + z^2)\hat{i} + (3xy^3 - x^2z)\hat{j} + (\lambda xy^2z + xy)\hat{k}$ is solenoidal.

Ex-11 Evaluate $\int_C (x^2 + y)ds$, where C is the straight line segment $x = 2t, y = 1 - t, z = 1$ for $0 \le t \le 1$.

Ex-12 If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}d\bar{r}$,where C is the arc of parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

Ex-13 Evaluate $\int_C (ydx + xdy + zdz) where C is x = \cos t, y = \sin t, z = t^2, 0 \le t \le 2\pi$.

Ex-14 Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ moves a partical in the XY plane from $O(0,0)$ to $P(1,1)$ along the parabola $x^2 = y$.

Ex-15 Let a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displace a partical in the XY plane from $O(0,0)$ to $P(1,4)$ along a curve $4x^2 = y$. Find work done.

Ex-16 Let $\vec{F} = 2x^2\hat{i} + xy\hat{j} + \hat{k}$ is the velocity field of a fluid in space. Find the flow along the curve $t\hat{i} + t\hat{j} + \hat{k}$ where $0 \le t \le 1$.

Ex-17 Show that the vector field $\vec{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$ is conservative and find the corresponding scalar potential.

Ex-18 Find a potential function for the field $\vec{F} = e^{y+2z}(\hat{i} + x\hat{j} + 2x\hat{k})$.

Ex-19 Show that $\vec{F} = (y^2z^3)\hat{i} + (2xyz^3)\hat{j} + (3xy^2z^2)\hat{k}$ is conservative vector field and find the corresponding potential function.

Ex- 20 Write the statement of Green's theorem in the plane.
Verify Green's theorem for the function $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ where C is the ractangle in the XY plane bounded by $y = 0, y = b, x = 0 \& x = a.$

Ex-21 Using Green's theorem , evaluate $\oint_C xydx + x^2y^3dy$,where C is the triangle with vertices $(0,0),(1,0),(1,2)$.

Ex-22 Verify Green's theorem for $\oint_C y^2dx + x^2dy$, where C is triangle bounded by $x = 0, x + y = 1 \& y = 0.$

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