Shantilal Shah Engineering College ,Bhavnagar

B.E. Sem-I (All Branches) **Sub : Mathematics-1(3110014)**

Tutorial :1 **Topic : Partial Derivatives**

Ex-1 Find the limit of following:

(a) $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$ (b) $\lim_{(x,y)\to(0,\pi/4)} \frac{e^y \sin x}{x}$

(c) $\lim_{(x,y)\to(\pi/2,0)} \frac{\cos y+1}{y-\sin x}$ (d) $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$ (e) $\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}, x\neq 1$

Ex-2 Show that $f(x,y)=\begin{cases}\frac{2xy}{x^2+y^2},(x,y)\neq(0,0)\\ 0,(x,y)=(0,0)\end{cases}$ is continuous at every point exept at the origin.

Ex-3 At what point (x,y) in the plane are the following function continuous ?

(a) $f(x,y)=\ln(x^2+y^2)$ (b) $f(x,y)=\sin\frac{1}{xy}$

Ex-4 By considering different paths of approach ,show that the following functions have no limit as $(x,y)\to(0,0)$.

(a) $f(x,y)=\frac{2x^2y}{x^2+y^2}$ (b) $f(x,y)=\frac{xy}{|xy|}$

Ex-5 State Sandwich theorem for two variables function,using it do the following.

(a) Suppose given $1-\frac{x^2y^2}{3}<\frac{\tan^{-1}xy}{xy}<1$ then find $\lim_{(x,y)\to(0,0)} \frac{\tan^{-1}xy}{xy}$.

(b) Suppose given $\left|\sin\frac{1}{x}\right|\le 1$ then find $\lim_{(x,y)\to(0,0)} y\sin\frac{1}{x}$

Ex-6 Find $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ for the following function:

(a) $f(x,y)=x^2+3xy+y-1$ at point (4,-5)

(b) $f(x,y)=\frac{2y}{y+\cos x}$

Ex-7 If resisters of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor,the value of R can be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$

. Find the value of $\frac{\partial R}{\partial R_1}$ and $\frac{\partial R}{\partial R_2}$ when $R_1=30, R_2=45$ & $R_3=90$ ohms.

Ex-8 Find the all second order partial derivatives(i.e. $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$ & $\frac{\partial^2 f}{\partial x \partial y}$)

of the following functions:

(a) $f(x,y) = x^2 y + \cos y + y \sin x$ (b) $g(x,y) = \tan^{-1}(y/x)$.

Ex-9 Evaluate $\frac{dw}{dt}$ at the given value of t for

(a) $w = x^2 + y^2, x = \cos t, y = \sin t, t = \pi.$

(b) $w = \frac{x}{z} + \frac{y}{z}, x = \cos^2 t, y = \sin^2 t, z = 1/t, t = 3.$

Ex-10 Find dy/dx (Implicit Differentiation) for the given below:

(a) $xy + y^2 - 3x - 3 = 0, (-1,1)$ (b) $y^2 - x^2 - \sin xy = 0.$

Ex-11 The ideal gas law states that if n moles of a gas has volume V and Temperature T and is under pressure p. Then pV=nkT , where k is universal gas constant ,then show that $\frac{\partial V}{\partial T}\frac{\partial T}{\partial p}\frac{\partial p}{\partial V} = -1.$

Ex-12 State Clairaut's Theorem:

If $u = \tan^{-1}(\frac{y}{x})$ verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Ex-13 State Euler's theorem on homogeneous function: Using it prove following

(a) If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$,show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$.

(b) If $u(x,y) = \tan^{-1}\left(x^2 + 2y^2\right)$,show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u$

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