

SHANTILAL SHAH ENGINEERING COLLEGE , BHAVNAGAR

BE Sem-II (All Branches) Mathematics-1 (3110015)

Tutorial-5 Fourier Integral , Series Solution (Ex-1 to 13)

Ex-1 Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral.

Hence evaluate $\int_0^{\infty} \frac{\sin \rho \cos \rho x}{\rho} d\rho$.

Ex-2 Using Fourier Sine integral, show that $\int_0^{\infty} \frac{1 - \cos \pi \rho}{\rho} \sin x \rho d\rho = \begin{cases} \frac{1}{2} \pi, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$.

Ex-3 Find the Fourier cosine integral of $f(x) = e^{-ax}; x > 0, a > 0$.

Ex-4 Find the Fourier integral of $f(x) = \begin{cases} 2; & |x| < 2 \\ 0; & |x| > 2. \end{cases}$

Ex-5 Find the power series solution near $x = 0$ of the equation $y'' + y = 0$.

Ex-6 Find the series solution of $y'' = 2y'$ in power of x

Ex-7 Use Frobenius method, solve $2x^2 y'' + xy' + (-1 + x^2)y = 0$.

Ex-8 Use Frobenius method, solve $x^2 y'' - xy' + (1 + x^2)y = 0$.

Ex-9 Express $2 - 3x + 4x^2$ in terms of Legendre's polynomials.

Ex-10 Express $x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of Legendre's polynomials

(By using Rodrigue's formula or by any known formula)

Ex-11 Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

Ex-12 Show that $J_1(x) = J_0(x) - x^{-1} J_1(x)$

Ex-13 Prove that $\frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x)$.
