

Shantilal Shah Engineering College ,Bhavnagar

B.E. Sem-I (All Branches)

Sub :Mathematics-1(3110014)

Tutorial :3

Topic: Indeterminate form, Beta-Gamma functions, Improper Integral

Ex-1 Solve the following:

- $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^2 \tan x}$
- $\lim_{x \rightarrow \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2ex}$
- $\lim_{x \rightarrow \frac{\pi}{2}} \frac{3 \sec x}{1 + \tan x}$
- $\lim_{x \rightarrow 0} \sin^2 \frac{x}{2} \ln x$
- $\lim_{x \rightarrow 0} \frac{1}{x} (1 - x \cot x)$
- $\lim_{x \rightarrow 0} (\sin x)^{\tan x}$
- $\lim_{x \rightarrow 0} \left(\frac{e^x + e^{2x} + e^{3x}}{3} \right)^{\frac{1}{x}}$

Ex-2 Define Beta and Gamma function and state relation between Beta and Gamma functions.

Ex-3 Evaluate the following :

- $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$, β denote Beta function
- $\int_3^7 \sqrt[4]{(x-3)(7-x)} dx$ by using Beta and Gamma functions
- Prove that $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$, β denote Beta function.
- $\beta(m+2, n-2) = \frac{m(m+1)}{(n-1)(n-2)} \beta(m, n); m, n > 0.$

Ex-4 Define Improper Integral first kind and second kind both.

Ex-5 Evaluate the following Improper Integrals:

- $\int_0^{\infty} \frac{1}{1+x^2} dx$
- $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx$
- Check the convergence of $\int_0^{\infty} \frac{1}{(1+x^2)(1+\tan^{-1} x)} dx$
- Show that $\int_0^{\infty} e^{-x^2} dx$ is convergent.