LP FORMULATION – ASSIGNMENT

1. A small manufacturer employs five skilled men and ten semi skilled men for making a product in two qualities: a deluxe model and an ordinary model. The production of a deluxe model requires 2-hours work by a skilled man and a 2-hour work by a semi skilled man. The ordinary model requires 1-hour work by a skilled man and a 3-hours work by a semi skilled man. According to worker union's rules, no man can work more than 8-hours per day. The profit of the deluxe model is Rs.1000 per unit and that of the ordinary model is Rs.800 per unit. Formulate a linear programming model for this manufacturing situation to determine the production volume of each model such that the total profit is maximized.

2. A firm manufactures three products A, B & C. Their profits per unit are Rs.300, Rs.200 and Rs.400, respectively. The firm has two machines and the required processing time in minutes on each machine for each product is given in the following table:

	Product A	B	C
Machine 1	4	3	5
Machine 2	2	2	4

Machines 1 and 2 have 2000 and 2500 machine minute, respectively. The upper limits for the production volumes of the product A, B & C are 100 units, 200 units and 50 units, respectively. But, the firm must produce a minimum of 50 units of the product A. Develop a LP model for this manufacturing situation to determine the production volume of each product such that the total profit is maximized.

3. The manager of an oil refinery has to decide on the optimal mix of two possible blending processes. The inputs and the outputs per production run of the blending process are as follows:

	Input		**Output**	
Process	Crude A	Crude B	Gasoline G_1	Gasoline G_2
1	5	3	5	8
2	4	5	4	4

The maximum amounts of availability of crude A and crude B are 200 units and 150 units, respectively. Market requirements show that at least 100 units of Gasoline G_1 and 80 units of Gasoline G_2 must be produced. The profits per production run from process 1 and process 2 are Rs.3 lacs and Rs. 4 lacs respectively. Formulate this problem as a LP model to determine the number of production runs of each process such that the total profit is maximized.

SOLVE EXAMPLES USING SIMPLEX METHOD

1. Minimize $Z=2x+9y-4z$
   Subject to $2x+3y+4z \leq 16$
   $x+6y-4z \geq 16$
   $x,y,z > 0$ [$S_1=8$, $y=8/3$, $Z=24$]
2. Maximize $z=4x+5y$
   Subject to $2x+3y \leq 8$
   $3x+y \geq 4$
   $x,y \geq 0$
   Use big M method. [$S_2=8$, $x=4$, $z=16$]
3. Maximize $Z=6x+10y+8z$
   Subject to $2x+3y \leq 8$
   $2y+5z \leq 10$
   $3x+2y+4z \leq 15$
   $x,y,z \geq 0$ [$y=750/41$, $z=930/41$, $x=89/41$, $Z=212.78$]
4. Maximize $Z=40x+30y$
   Subject to $3x+2y \leq 300$
   $x+y \leq 80$
   $2x+y \leq 200$
   $3x+4y \leq 300$
   $x \leq 60$
   $y \leq 60$
   $x,y \geq 0$ [$S_1=80$, $y=20$, $S_3=60$, $S_4=40$, $x=60$, $S_6=40$, $Z=3000$]
5. Minimize $Z=20x_1+10x_2$
   Subject to $x_1+2x_2 \leq 40$
   $3x_1+x_2=30$
   $4x_1+3x_2 \geq 60$
   $x_1, x_2 \geq 0$
   Use (i) Big 'M' Method.
   (ii) Two – Phase Method [$x_1=6$, $x_2=12$, $S_1=10$, $Z=240$]
6. Solve the following problem:
   Maximize $Z= -x_1-4x_2-3x_3$
   Subject to $2x_1+x_2+3x_3 \geq 4$
   $x_1+2x_2+2x_3 \geq 3SS$
   $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign. [Problem is infeasible]

FORMULATE PROBLEM AND SOLVE USING SIMPLEX METHOD

1. Shri Rajendra Desai has booked orders for product A and product B at the sales price of Rs. 6 and Rs. 8 per unit respectively, for M/S supreme Engineers with the condition that materials cost will be borne by the company i.e. M/S Supreme Engineers. Shri Desai negotiates the terms with the Kartik Engineering, a job order shop, which can offer machines of type x for 60 hours per week and machines of type y for 100 hours per week at hourly rates. Each machine can produce product A and B in any combination.

   Machine-hours of type x are available with the operators of the Kartik Engineering only and the rates are Rs. 4 per machine-cum-operator-hour. Machines of type y are available with two alternative offers viz. Desai can either hire the machines alone at the rate of Rs. 1/machine hour as well as operator at the rate of Rs. 3 per hour or Desai can hire his own operator for machines y at the rate of Rs. 3 per hour. The production rates are given in the Exhibit A.

Production rate, units per hour Exhibit – A

Product	Machines, Type x	Machines, Type y	
		With operator of Kartik Engg.	With Desai's operator
A	10	6	8
B	20	12	18

Formulate L.P.P. to maximize profit of Shri Desai.

[Maximize

$Z = (6- 4/10) X_1 + (8-4/20) X_2 + (6-4/6) X_3 + (8-4/12)X_4 +(6-4/8)X_5 + (8-4/18)X_6$

Subject to $X_1 /10 + X_2 /20 \leq 60$

$X_3 /6 + X_4 /12 + X_5 /8 + X_6 /18 \leq 10$

$X_1, X_2, X_3, X_4, X_5, X_6, \geq 0$]

2. The State Industries Finance Corporation is to decide its investment of Rupees 20 lacs on two proposals of Private sector Industries X and Y> X undertakes to guarantee annual return of 10 per cent, and Y of 15 per cent. As X is in the line of basic consumer products and that too co-operative venture, the Government has laid down that at least Rs.5 lacs should be invested in X. The corporation would like to have investments so made as to procure the minimum of 12 per cent annual return. For the reason best known to one of the Directors, following his rigid attitude in this regard SIFC has decided not to have investment in Y and X more than in the ratio 1.6: 1. Formulate this L.P. problem for maximum annual return. Solve it be either (1) Graphical method or (2) Simplex method.

[maximize

$Z = 0.1 X_1 + 0.15 X_2$

Subject to $X_1 + X_2 = 20$

$0.1 X_1 + 0.15 X_2 \geq 0.12 (X1 + X_2)$

$X_1 \geq 5$

$X_2 / X_1 \leq 1.6 / 1$

$X_1, X_2 \geq 0$

$X_1 = 765/91, X_2 = 1055/91, S_2 = 107/260, Z = 2.5797$]

3. The following data is available for two products.

Product	Production Alternatives	Production Rate in units/hr.	
		Centre 1	Centre 2
1	1	0.4	---------
	2	----------	0.5
2	1	0.5	1.0
Production hrs. available/week		500	400
Production cost Rs./hr.		6	8

Material cost is same for both the products and is Rs. 5 per unit. Selling price Rs. Per unit for product one is 25 and that of product two is 31. Solve the problem for optimal product mix.

[$X1 = 1250, X_2 = 800, X_3 =0, Z = 9450$]

4. A plant can produce three products, A, B and C, Product A needs 4 hours of department 2 and one hour of department 3. Product B needs two hours of department 1 and two hours of department 2, while product C needs two hours of department 1 and two hours of department 3. Their respective profit coefficients in Rs. Per unit are 50, 40, and 55. Hours available per month with department 1, 2, and 3 are 1000, 1000, and 800 respectively.

   A purchased part is used in assembly of product A and C. Only 4000 parts are available for the month. Two such parts are used in each piece of product A and three parts in each piece of product C. There is a sales commitment of 200 units of product A. Find the optimal product mix.

DUALITY & SENSITIVITY ANALYSIS

Problem 1:

Construct the dual of the problem:

Maximize $Z = 3x+5y$

Subject to $X- 2y \geq 3$

$X + 3y \geq 9$

$X – y \leq 5$

$X \geq 0$, is unrestricted in sign

Solve the dual.

[DUAL IS MAXIMIZE $Z = -3A-9B+5C$
SUBJECT TO $–A-B+C \geq 3$
$-2A+3B+C \geq -5$
$A,B,C \geq 0$.
$C=11, A=8, S3=0, Z=31$]

Problem: 2

Use dual simplex method to solve the following

Minimize $Z = 2x + 3y$

Subject to $3x + 4y \geq 5$

$4x + 5y \geq 7$

$x + 2y \leq 4$

$x , y \geq 0$

[S1 = ¼, X= 7/4, S3 =9/4, Z = 7/2]

Problem: 3

The following tableau gives an optional solution to a standard linear programme:

Maximize $Z = CX$.

Subject to $AX = B, X \geq 0$

Tableau			5	7	-4	0	0
			X1	X2	X3	S1	S2
7	x2	x3	¾	1	-1/4	¼	0
0	s2	½	¼	0	-3/4	-5/4	1
$(z_j – c_j)$		7/2	¼	0	9/4	7/4	0

S_1 and S_2 are the slack variables.

(a) How much can c_1 be increased before the current basis is no longer optimal?

(b) How much can c_2 be varied sp that the given basis (x_2 , S_2) is still optimal?

(c) What are the shadow prices of two resources?

[(a) $c_1=21/4$, $x_1=2/3$, $S_2=1/3$, $Z=4$ (b) $20/3 \leq c_2 \leq 16$ (c) 7/4 and 0]

QUING THEORY

1. Customer arrivals at a teller counter in a Commercial Bank are considered to be following Poisson probability distribution with an average time of 10 minutes between one arrival and the next. The time length of service that is rendered is assumed to be distributed exponentially with mean three minutes
    i) What is the probability that a person arriving at the teller will have to wait?
    ii) What is the average length of queue that forms from time to time?
    iii) The Commercial Bank will install a second teller man when convince that the arrival rate increase in order to justify the second booth. When does this happen?
2. There is congestion on the platform of Ahmed Railway station. The trains arrive at the rate of 30 trains per day. The waiting time for any train to flag-off is exponentially distributed with an average of 36 minutes. Calculate the following:
    i) The mean queue size.
    ii) The probability that the queue size exceeds 10.

INVENTORY CONTROL ASSIGNMENT

1. The Bombay Shoe Company has found that it purchases a large amount of industrial tapes for production of its shoes. Currently, it purchases Rs.10,00,000 a year of the various sized tapes from the local manufactures. A proposal was made by its supplier, was offer consist of 1.25% discount if BSC places an order quarterly BSC has calculated the cost to purchase at Rs.22.5 per order. Rs.20 per tape & inventory carrying costs at 18%. Should BSC accept discount offer from its supplier? If not, what alternate offer should be made in term of a discount?
2. The Himavan manufacturing company wishes to determine the most economy quantity for one of its products. Manufacturing cost amount to Rs.15 per unit. The production is 5000 units per annum. Each lot now requires a set-up cost of Rs.25 and the inventory cost of 25% of the average inventory value. What is the most economic lot size to manufacturers? What is the corresponding total yearly cost?
3. Demand for a certain part order by Shah brothers tends to be constant at the monthly rate of 1000 units. The per unit carrying cost of this item is Rs.25 per year, & cost of placing an order is Rs.75.
    i) What is the optimal order size? How often should an order be placed?
    ii) Show that the annual holding cost & ordering cost are equal when an optimal order size is used. What is the total relevant cost?
    iii) If the company wants to order only one every other week, by what percentage would this increase the total relevant cost?
4. The Calcutta Tool Company can manufacture a certain type of tool at the rate of 1480 per run, the demand for this tool is quite steady at annual rate of 9000 units. Unit cost of the tool is Rs.30, an set up cost per production run is Rs.500. The annual cost per unit sort is Rs.20. The company has determined the optimal production lot size to be 3000 units.
    i) What is the annual inventory holding cost, expressed as percentage of unit cost?
    ii) What is the total relevant cost & the max. inventory level?
5. Let C1=50(Rs.), Cs=20C1, & the demand distribution is as under:

r	0	1	2	3	4	5	6
P(r)	0.9	0.05	0.02	0.01	0.01	0.01	0

What is the optimum quantity to be ordered?

5. Assume that demand during a certain time interval T is random with P(r) being the probability that the total demand is r during interval T. The demand rate is constant during the interval T.
C1=10(Rs.), C2=20C1 and the demand distribution is as under:

r	0	1	2	3	4	5	6
P(r)	0.1	0.2	0.2	0.3	0.1	0.1	0

What should be the initial inventory level?

PROBLEMS ON GAMES THEORY

1. Linear programming method:

(i)

		B		
		1	2	3
	1	1	7	2
A	2	6	2	7
	3	6	1	6

(ii)

		B		
		1	2	3
A	1	5	7	9
	2	4	8	11

(iii)

		B		
		1	2	3
A	1	-1	6	4
	2	5	-3	-6

(iv)

		B		
		1	2	3
	1	4	7	9
A	2	11	6	10
	3	5	6	9

6

(v)

		B		
		1	2	3
	1	-5	10	20
A	2	5	-10	-10
	3	5	-20	-20

(vi)

		B			
		1	2	3	4
	1	3	-1	1	2
A	2	-2	3	2	3
	3	2	-2	-1	1

(vii)

		B			
		1	2	3	4
	1	4	10	11	14
	2	15	5	13	18
A	3	4	9	6	10
	4	17	12	10	7
	5	14	18	17	8
	6	11	14	15	9

(viii)

		B			
		1	2	3	4
	1	-1	-3	5	4
A	2	-3	4	-3	-2
	3	-3	-2	4	3

TRANSPORTATION

Problem: Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method:

	Warehouses				
	W_1	W_2	W_3	W_4	Capacity
F_1	10	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	34

Problem: A company has received a contract to supply gravel for three new construction projects located in towns A, B and C. Construction engineers have estimated the required amounts of gravel which will be needed at these construction projects as shown below:

Project location	Weekly requirement (Truck loads)
A	72
B	102
C	41

The company has three gravel plants X, Y and Z located in three different towns. The gravel required by the construction projects can be supplied by these three plants. The amount of gravel which can be supplied by each plant is as follows:

Plant	Amount available/week (Truck loads)
X	76
Y	62
Z	77

The company has computed the delivery cost from each plant to each project site. These costs (in rupees) are shown in the following table:

		Cost per load		
		A	B	C
Plant	X	4	8	8
	Y	16	24	16
	Z	8	16	35

(a) Schedule the shipment from each plant to each project in such a manner so as to minimize the total transportation cost within the constraints imposed by plant capacities and project requirements.
(b) Find the minimum cost.
(c) Is the solution unique? If it is not, find alternative schedule with the same minimum cost.

ASSIGNMENT

Problem: Solve the following assignment problem using Hungerian method. The matrix entries are processing times in hours.

		Operator				
		1	2	3	4	5
Job	1	20	22	35	22	18
	2	4	26	24	24	7
	3	23	14	17	19	19
	4	17	15	16	18	15
	5	16	19	21	19	25

Problem: Consider the problem of assigning four sales persons to four different sales regions as shown below such that the total sales is maximized.

		Sales region			
		1	2	3	4
Salesman	1	5	11	8	9
	2	5	7	9	7
	3	7	8	9	9
	4	6	8	11	12

The cell entries represent annual sales figures in crores of rupees. Find the optimal allocation of the sales persons to different regions.

Problem: The flight timings between two cities, X and Y are as given in the following two tables. The minimum layover time of any crew in either of the cities is 3 hours. Determine the base city for each crew so that the sum of the layover times of all the crews in non-base cities is minimized.

Timing of Flights form City X to City Y

Flight number	Departure time (from City X)	Arrival time (to City Y)
101	6 a.m.	8.00 a.m.
102	10 a.m.	12.00 noon
103	3 p.m.	5.00 p.m.
104	8 p.m.	10.00 p.m.

Timing of Flights from City Y to City X

Flight number	Departure time (from City Y)	Arrival time (to City X)
201	5.30 a.m.	7.00 a.m.
202	9.00 a.m.	10.30 a.m.
203	4.00 p.m.	5.30 p.m.
204	10.00 p.m.	11.30 p.m.

NETWORK PROBLEMS:

Problem: Consider the details of a distance network as shown below:

Arc	Distance	Arc	Distance
1-2	8	3-6	6
1-3	5	4-5	8
1-4	7	4-6	12
1-5	16	5-8	7
2-3	15	6-8	9
2-6	3	6-9	15
2-7	4	7-9	12
3-4	5	8-9	6

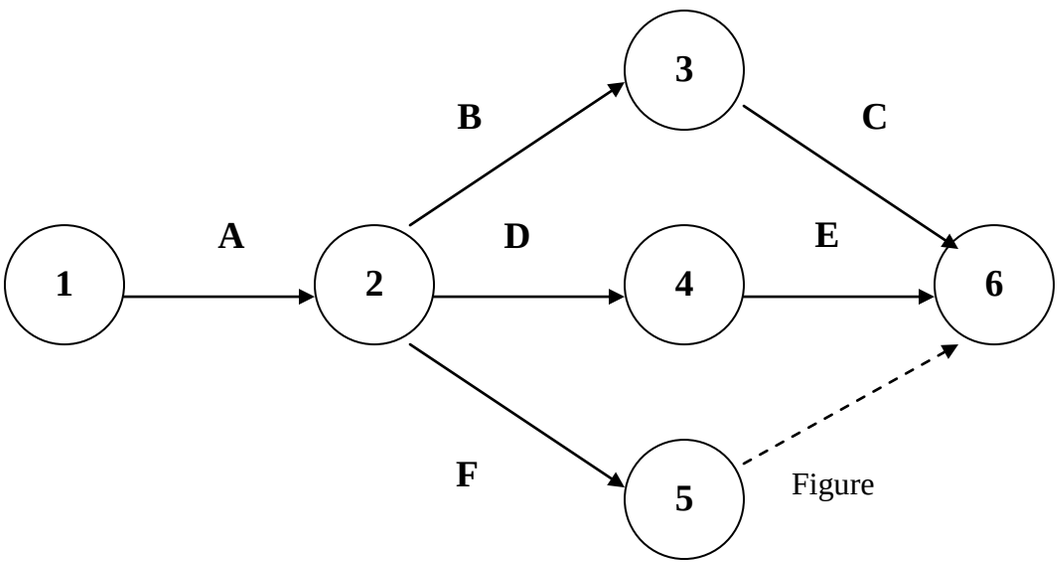
(a) Construct the distance network.
(b) Find the shortest path from Node 1 to Node 9 using the systematic method.
(c) Find the shortest path from Node 1 to Node 9 using Dijkstra's algorithm.

Problem: Consider the details of a distance network as shown below:

Arc	Distance
1-2	3
1-3	8
1-4	10
2-3	4
2-4	7
3-4	2
3-5	8
3-6	6

(a) Construct the detail network.
(b) Apply Floyd's algorithm and obtain the final matrices, D^5 and P^5.
(c) Find the shortest path and the corresponding distance for each of the following:
  (i) from Node 1 to Node 5
  (ii) from Node 2 to Node 5.

Problem: For the network shown in the Figure activity, resource requirements are given in the table. Solve the problem for resource allocation. There are two cranes and 8 welders available for the project.



Figure

Table

Activity	**A**	**B**	**C**	**D**	**E**	**F**
Duration Days	10	10	8	14	3	9
Resources Required	-	2 Cranes	8 Welders	6 Welders	-	1 Crane

Problem: The critical path network analysis is given in Figure. The activity times are also given for each activity. Determine the critical path and critical path time. Also determine the floats for each activity.

Figure