ASSIGNMENT NO - 2
Tutorials based on LP Model Formulation

Examples on Production

1. A company has two grades of inspectors 1 and 2, who are to be assigned for a quality control inspection. It is required at least 2000 pieces be inspected per 8 hour day. A grade 1 inspector can check pieces at the rate of 40 per hour, with an accuracy of 97 %. A grade 2 inspector checks at the rate of 30 pieces per hour with an accuracy of 95 %. The wage rate of grade 1 inspector is Rs 7 per hour while that a grade 2 inspector is Rs 6 per hour. An error made by an inspector costs Rs 4 to the company. There are only nine grade 1 inspectors and eleven grade 2 inspectors available in the company. The company wishes to assign work to the available inspectors so as to minimize the total cost of the inspection. Formulate this problem as a linear programming model.

2. A manufacturing company is engaged in producing three types of products: A, B and C. The production department daily produces components sufficient to make 60 units of A, 30 units of B and 35 units of C. The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The following additional information is available:

Type of products	**Profit contribution per unit of product (Rs)**	**Assembly time per product (hrs)**
A	**12**	**0.8**
B	**20**	**1.7**
C	**45**	**2.5**

The company has a daily order commitment for 20 units of product A and total of 15 units of B and C product. Formulate this problem as an LP Model so as to maximize the total profit.

3. A company has two plants each of which produces and supplies two products: A and B. The plants can each work up to 16 hours a day. In plan 1, it takes 3 hours to prepare and pack 1000 gallons of A and 1 hour to prepare and pack one quintal of B. In plant 2, it takes 2 hours to prepare and pack 1000 gallons of A and 1.5 hours to prepare and pack a quintal of B. In plant 1, it costs Rs 15,000 to prepare and pack 1000 gallons of A and Rs 28,000 to prepare and pack a quintal of B whereas these costs are Rs 18,000 and Rs 26,000 respectively in plant 2. The company is obliged to produce daily at least 10 thousand gallons of A and 8 quintals of B. Formulate this problem as an LP model to find out as to how the company should organize its production so that the required amounts of the two products be obtained at minimum cost.

4. A firm manufactures two product A & B on which the profit earned per unit are Rs. 3 and Rs.4, respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2, while product B requires one minute of processing time on M1 and one minute on M2. Machine M1 is

available for not more than 7 hrs and 30 minutes, while machine M2 is available for 10 hrs during any working day. Formulate this problem as LPP.

5. A small fabrication industry is faced with a problem of scheduling production and subcontracting for three products A, B and C. Each product requires casting, machining and assembly operations. Casting operation for product A and B can be subcontracted but product C requires special tooling hence it cannot be subcontracted. Each unit of product A, B and C requires 6, 10 and 8 minutes of casting time in the foundry shop of a company. Machining times per unit of products A, B and C are 6, 3 and 8 minutes while assembly times are 3, 2 and 2 minutes respectively. The time available per week in foundry, machining and assembly shop are 8000, 12000 and 10000 minutes respectively. If product A, B and C are produced completely in the company, the overall profits per unit of product are Rs. 700, Rs. 1000 and Rs. 1100 respectively. When castings are obtained from subcontractors, the profit per unit of product A and B are Rs. 500 and 900 respectively. Formulate above problem as LPP so as to maximize the profit for company by scheduling its production and subcontracting.

6. A coffee company mixes Brazilian, Columbian and African coffee to make two brands of coffee plains A and B. The characteristics used in blending the coffee include strength, acidity and caffine. The test result of the available supply of Brazilian, Columbian and African coffee.

	Price/kg	Strength	Acidity	%coffine	Supply available
Brazilian	60	6	4	2	50000
Columbian	70	8	3	2.5	30000
African	65	5	3.5	1.5	25000

The requirement for A and B coffee are given as below.

Plain coffee	Price/kg	Min strength	Max acididty	Max % coffine	Quantity Demanded
A	75	6.5	3.8	2.2	65000
B	85	6.0	3.5	2	55000

Assume that 35000 kg of plains A and 25000 kg of plain B are to be sold formulate LPP.

Examples on Marketing

1. An advertising company wishes to plan an advertising campaign in three different media: Television, Radio and Magazines. The purpose of the advertising is to reach as many potential customers as possible. Results of a market study are given below:

Data Items	Types of Media			
	Television		Radio	Magazines
	Prime day (Rs)	Prime Time (Rs)	(Rs)	(Rs)
Cost of an advertising unit	40,000	75,000	30,000	15,000
Number of potential customers reached per unit	4,00,000	9,00,000	5,00,000	2,00,000
Number of women customers reached per unit	3,00,000	4,00,000	2,0,000	1,00,000

The company does not want to spend more than Rs 8, 00,000 on advertising. It is further required that

1) At least 2 million exposures take place among women,
2) Advertising on television be limited to Rs 5,00,000
3) At least 3 advertising units be bought on prime day and two units during prime time; and
4) The number of advertising units on radio and magazine should each be between 5 and 10 Formulate this problem as LP Model.

2. An advertising agency is preparing an advertising campaign for a group of agencies. These travel agencies have decided that their target customers should have the following characteristics with importance (weight age) as given below:

Data Items	Characteristics	Weight age (%)
Age	25 – 40 years	20
Annual Income	Above Rs 60,000	30
Female	Married	50

The agency has made a careful analysis of three media and has complied the following data:

Data Items	Types of Media		
	Women's magazine (%)	Radio (%)	Television (%)
Readers characteristics :			
(i) Age : 25 – 40 years	60	70	80

(ii) Annual income : Above Rs 20,000	60	50	45
(iii) Female / Married	40	35	25
Cost per advertisement (Rs)	9,500	25,000	1,00,000
Minimum number of advertisement allowed	10	5	5
Maximum number of advertisement allowed	20	10	10
Audience size (1000's)	750	1,000	1,500

The budget for launching the advertising campaign is of Rs 5,00,000. Based on the available data, formulate this problem as LP Model for the agency to maximize the expected effective exposures.

Examples on Finance

1. An engineering company is planning to diversify its operations during the year 1996-97. The company has allocated capital expenditure budget equal to Rs 5.15 Crore in the year 1996 and Rs 6.50 crore in the year 1997. The company has five investment projects under consideration. The estimated net returns at present value and expected cash expenditures of each project in the two years are as follows:

Project	Estimated net returns (in '000 Rs.)	Cash expenditure (in '000 Rs.)	
		Year 1996	Year 1997
A	240	120	320
B	390	550	594
C	80	118	202
D	150	250	340
E	182	324	474

Assume that the returns from a particular project would be in direct proportion to the investment in it, so that, for example, if in a project, say A, 20% (of 120 in 1996 and of 320 in 1997) is invested, then the resulting net returns in it would be 20% (of 240). This assumption also implies that individuality of the project should be ignored. Formulate this capital budgeting problem as an LP model to maximize the net returns.

2. An investor has investment opportunities available at the beginning of each of the next 5 years, and also has a total of Rs 5,00,000 available for investment at the beginning of the first year. A summary of the financial characteristics of the three investment alternatives is presented in the following table :

Investment alternative	Allowable size of initial investment (Rs.)	Return (%)	Timing of return	Immediate reinvestment possible ?
1	1,00,000	19	1 year later	Yes
2	Unlimited	16	2 years later	Yes
3	50,000	20	3 years later	Yes

This investor wishes to determine the investment plan that will maximize the amount of money which can be accumulated by the beginning of the 6^{th} year in the future. Formulate this problem as an LP model so as to maximize total return.

Example on Agriculture

1. A co-operative farm owns 100 acres of land and has Rs. 25,000 in funds available for investment. The farm members can produce a total of 35000 man – hours worth of labour during the months September – May and 4000 man – hours during June – August. If any of these man – hours are not needed, some members of the firm will use them to work on a neighbouring farm for Rs 2 per hour during September – May and Rs 3 per hour during June – August. Cash income can be obtained from the three main crops and two types of livestock: dairy cows and laying hens. No investment funds are needed for the crops. However, each cow will require an investment outlay of Rs 3200 and each hen will require Rs 15.

   Moreover, each cow will require 1.5 acres of land, 100 man-hours during September – May and 50 man-hours during June-August. Each cow will produce a net annual cash income of Rs 3500 for the farm. The corresponding figures for each hen are : no acreage, 0.6 man-hours during September – May; 0.4 man-hours during June-August, and an annual net cash income of Rs 200. The chicken house can accommodate a maximum of 4000 hens and the size of the cattle-shed limits the number to a maximum of 32 cows. Estimated man-hours and income per acre planted in each of the three crops are :

Particular	Paddy	Bajra	Jowar
Man-hours			
September – May	40	20	25
June-August	50	35	40
Net annual cash income (Rs)	1200	800	850

Formulate this problem as an LP model so as to maximize total cash income.

Example on Personnel

1. A machine tool company conducts a job training programme for machinist. Trained machinists are used as teachers in the programme at the ratio of one for every ten trainees. The training programme lasts for one month. From past experience it has been found that out of ten trainees hired, only seven complete the programme successfully and the rest are released. Trained machinists are also needed for machining and company's requirements for the next three months are as follows: January 100, February 150 and March 200. In addition, the company requires 250 machinists by April. There are 130 trained machinists available at the beginning of the year. Payroll per month are :

Each trainee	Rs 1400
Each trained machinist (machining and teaching)	Rs 1900
Each trained machinist idle	Rs 1700

Formulate this problem as an LP model so as to minimize the cost of hiring and training schedule and the company's requirements.

Q.1. Define the following terms in context with Linear Programming Model :

1. Linear Programming
2. Decision variables
3. Constraints

Q.2. Explain the structure and assumptions of LP Model.

Q.3. What are the assumptions of LP Model ? Explain.

Q.4. Give the mathematical structure of linear programming problem. What requirements should be met in order to apply linear programming ?

Q.5. "Linear Programming is one of the most frequently and successfully applied mathematical approaches to managerial decisions." Comment.

Q.6. State the procedure of linear programming model formulation.

Reference Books :
Operation Research By J K Sharma
Operation Research By V K Kapoor
Operation Research By Hamdy A. Taha

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ASSIGNMENT NO - 3
Tutorials based on LP Graphical Method

Q. 1 Define the following terms in context with LP :

1 Solution , 2. Feasible Solution, 3. Basic Solution, 4. Basic Feasible Solution,
5. Degenerate Basic Feasible Solution, 6. Non-degenerate Basic Feasible Solution
7. Optimum Basic Feasible Solution.

Q. 2 State the procedure of Graphical method to solve the LP Problem.

Use the graphical method to solve the following LP Problems:

1. Maximize $Z = 15X_1 + 10X_2$
   Subject to constraints
   $$4X_1 + 6X_2 \leq 360$$
   $$3X_1 + 0X_2 \leq 180$$
   $$0X_1 + 5X_2 \leq 200$$
   $$X_1, X_2 \geq 0$$

2. Maximize $Z = 2X_1 + X_2$
   Subject to constraints
   $$X_1 + 2X_2 \leq 10$$
   $$X_1 + X_2 \leq 6$$
   $$X_1 - X_2 \leq 2$$
   $$X_1 - 2X_2 \leq 1$$
   $$X_1, X_2 \geq 0$$

3. Maximize $Z = 10X_1 + 15X_2$
   Subject to constraints
   $$2X_1 + X_2 \leq 26$$
   $$2X_1 + 4X_2 \leq 56$$
   $$-X_1 + X_2 \leq 5$$
   $$X_1, X_2 \geq 0$$

4. Minimize $Z = 3X_1 + 2X_2$
   Subject to constraints
   $$5X_1 + X_2 \geq 10$$
   $$X_1 + X_2 \geq 6$$
   $$X_1 + 4X_2 \geq 12$$
   $$X_1, X_2 \geq 0$$

5. Minimize $Z = -X_1 + 2X_2$
   Subject to constraints
   $$-X_1 + 3X_2 \geq 10$$
   $$X_1 + X_2 \geq 6$$
   $$X_1 - X_2 \geq 2$$
   $$X_1, X_2 \geq 0$$

6. Minimize $Z = 200X_1 + 400X_2$
   Subject to constraints
   $$X_1 + X_2 \geq 200$$
   $$X_1 + 3X_2 \geq 400$$
   $$X_1 + 2X_2 \leq 350$$
   $$X_1, X_2 \geq 0$$

7. Maximize $Z = 7X_1 + 3X_2$
   Subject to constraints
   $$X_1 + 2X_2 \geq 3$$
   $$X_1 + X_2 \leq 4$$
   $$0 \leq X_1 \leq 5/2$$
   $$0 \leq X_2 \leq 3/2$$
   $$X_1, X_2 \geq 0$$

8. Minimize $Z = 20X_1 + 10X_2$
   Subject to constraints
   $$X_1 + 2X_2 \leq 40$$
   $$3X_1 + X_2 \geq 30$$
   $$4X_1 + 3X_2 \geq 60$$
   $$X_1, X_2 \geq 0$$

9. Maximize $Z = 2X_1 + 3X_2$
   Subject to constraints
   $$X_1 + X_2 \leq 30$$
   $$0X_1 + X_2 \geq 3$$

10. Maximize $Z = 80X_1 + 120X_2$
    Subject to constraints
    $$X_1 + 2X_2 \leq 9$$
    $$X_1 \geq 2$$

$$0 \le X_1 \le 20$$
$$0 \le X_2 \le 12$$
$$X_1 - X_2 \ge 0$$
$$X_1,\ X_2 \ge 0$$

$$X_2 \ge 3$$
$$20X_1 + 50X_2 \le 360$$
$$X_1,\ X_2 \ge 0$$

11. Minimize $Z = 5X_1 + 8X_2$

Subject to constraints

$$X_1 \le 4$$
$$X_2 \ge 2$$
$$X_1 + X_2 = 5$$
$$X_1,\ X_2 \ge 0$$

12. Maximize $Z = 300X_1 + 400X_2$

Subject to constraints

$$5X_1 + 4X_2 \le 200$$
$$3X_1 + 5X_2 \le 150$$
$$5X_1 + 4X_2 \ge 100$$
$$8X_1 + 4X_2 \ge 80$$
$$X_1,\ X_2 \ge 0$$

13. Maximize $Z = 50X_1 + 30X_2$

Subject to constraints

$$2X_1 + X_2 \ge 18$$
$$X_1 + X_2 \ge 12$$
$$3X_1 + 2X_2 \le 34$$
$$X_1,\ X_2 \ge 0$$

Some Special Cases in Linear Programming Model

Define and explain the following terms :

1. Alternative or Multiple Optimal Solution
2. An Unbounded Solution
3. An Infeasible Solution

14. Maximize $Z = 10X_1 + 6X_2$

Subject to constraints

$$5X_1 + 3X_2 \le 30$$
$$X_1 + 2X_2 \le 18$$
$$X_1,\ X_2 \ge 0$$

15. Maximize $Z = 3X_1 + 4X_2$

Subject to constraints

$$X_1 - X_2 \le -1$$
$$-X_1 + X_2 \le 0$$
$$X_1,\ X_2 \ge 0$$

16. Maximize $Z = 3X_1 + 2X_2$

Subject to constraints

$$X_1 - X_2 \le 1$$
$$X_1 + X_2 \ge 3$$
$$X_1,\ X_2 \ge 0$$

17. Maximize $Z = 6X_1 - 4X_2$

Subject to constraints

$$2X_1 + 4X_2 \le 4$$
$$4X_1 + 8X_2 \ge 16$$
$$X_1,\ X_2 \ge 0$$

18. Maximize $Z = X_1 + X_2/2$

Subject to constraints

$$3X_1 + 2X_2 \le 12$$
$$5X_1 \le 10$$
$$X_1 + X_2 \ge 8$$
$$-X_1 + X_2 \ge 4$$
$$X_1,\ X_2 \ge 0$$

19. A furniture manufacturer makes two products: chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hours on machine A and 6 hours on

machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours per day available on machine A and 30 hours on machine B. Profit gained by the manufacturer from a chair and a table is Rs. 2 and Rs. 10 respectively. What should be daily production of each of the two products ?

20. Company makes two kind of leather belts. Belt P is a high quality belt, and belt Q is of lower quality. The respective profits are Rs. 4 and Rs. 3 per belt. Each belt of type P requires twice as much time as a belt of type Q, and if all the belts were of type Q, the company could make 1200 per day. The supply of leather is sufficient for only 900 belts per day. Belt P requires a fancy buckle and only 390 per day are available. There are only 750 buckles a day available for belt Q. How should the company manufacture two types of belts in order to have a maximum overall profit ?

Reference Books :

Operation Research By J K Sharma
Operation Research By V K Kapoor
Operation Research By Hamdy A. Taha

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ASSIGNMENT NO - 4
Tutorials based on LP Simplex Method

Q.1. Define the following terms in context with LP Simplex Method :

a) Slack variable
b) Surplus variable
c) Artificial variable
d) Key element

Q.2. State the importance of key column, key row and index row to solve the LP Problem.

Q.3. State the steps of simplex algorithm to obtain an optimal solution to a standard linear programming problem for maximization case.

Q.4. What is degeneracy in simplex method ? How to resolve it ?

Solve the following LP Problems by using Simplex Method

1. Maximize $Z = 3X_1 + 5X_2 + 4X_3$

Subject to constraints

$$2X_1 + 3X_2 \leq 8$$
$$2X_2 + 5X_3 \leq 10$$
$$3X_1 + 2X_2 + 4X_3 \leq 15$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 89/41$, $X_2 = 50/41$, $X_3 = 62/41$, Max $Z = 765/41$

2. Maximize $Z = 3X_1 + 2X_2$

Subject to constraints

$$X_1 + X_2 \leq 4$$
$$X_1 - X_2 \leq 2$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 3$, $X_2 = 1$, and Max $Z = 11$

3. Maximize $Z = 5X_1 + 3X_2$

Subject to constraints

$$X_1 + X_2 \leq 2$$
$$5X_1 + 2X_2 \leq 10$$
$$3X_1 + 8X_2 \leq 12$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 2$, $X_2 = 0$, and Max $Z = 10$

4. Maximize $Z = 3X_1 + 2X_2 + 5X_3$

Subject to constraints

$$X_1 + 2X_2 + X_3 \leq 430$$
$$3X_1 + 2X_3 \leq 460$$
$$X_1 + 4X_3 \leq 420$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 0$, $X_2 = 100$, $X_3 = 230$, Max $Z = 1350$

5. Minimize $Z = X_1 - 3X_2 + 2X_3$

Subject to constraints

$$3X_1 - X_2 + 3X_3 \leq 7$$
$$-2X_1 + 4X_2 \leq 12$$
$$-4X_1 + 3X_2 + 8X_3 \leq 10$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 4$, $X_2 = 5$, $X_3 = 0$ and Max $Z^* = -Z = 11$

6. Maximize $Z = 4X_1 + 5X_2 + 9X_3 + 11X_4$

Subject to constraints

$$X_1 + X_2 + X_3 + X_4 \leq 15$$
$$7X_1 + 5X_2 + 3X_3 + 2X_4 \leq 120$$
$$3X_1 + 5X_2 + 10X_3 + 15X_4 \leq 100$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Ans. $X_1 = 50/7$, $X_2 = 0$, $X_3 = 55/7$ and Max $Z = 695/7$

7. Maximize $Z = X_1 + X_2 + X_3$

Subject to constraints

$$4X_1 + 5X_2 + 3X_3 \leq 15$$
$$10X_1 + 7X_2 + X_3 \leq 12$$
$$X_1, X_2, X_3 \geq 0$$

8. Maximize $Z = X_1 + X_2 + X_3$

Subject to constraints

$$3X_1 + 2X_2 + X_3 \leq 3$$
$$2X_1 + X_2 + 2X_3 \leq 2$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 0, X_2 = 0, X_3 = 5$ and Max $Z = 5$

Ans. $X_1 = 0, X_2 = 0, X_3 = 1$ and Max $Z = 3$

9. Maximize $Z = 2X_1 + 4X_2 + 3X_3 + X_4$
Subject to constraints

$$X_1 + 3X_2 + 0X_3 + X_4 \leq 4$$
$$2X_1 + X_2 + 0X_3 + 0X_4 \leq 3$$
$$0X_1 + X_2 + 4X_3 + X_4 \leq 3$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Ans. $X_1 = 1, X_2 = 1, X_3 = 1/2, X_4 = 0$ and Max $Z = 13/2$

10. Maximize $Z = 107X_1 + X_2 + 2X_3$
Subject to constraints

$$14X_1 + X_2 - 6X_3 + 3X_4 \leq 7$$
$$16X_1 + 1/2X_2 + 6X_3 \leq 5$$
$$3X_1 - X_2 - X_3 \leq 0$$
$$X_1, X_2, X_3, X_4 \geq 0$$

11. Maximize $Z = 4X_1 + 10X_2$
Subject to constraints

$$2X_1 + X_2 \leq 50$$
$$2X_1 + 5X_2 \leq 100$$
$$2X_1 + 3X_2 \leq 90$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 0, X_2 = 0$, and Max $Z = 200$

12. Maximize $Z = X_1 - X_2 + 3X_3$
Subject to constraints

$$X_1 + X_2 + X_3 \leq 10$$
$$2X_1 - X_3 \leq 2$$
$$2X_1 - 2X_2 + 3X_3 \leq 0$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 0, X_2 = 6, X_3 = 4$, and Max $Z = 6$

13. Maximize $Z = 4X_1 + X_2 + 3X_3 + 5X_4$
Subject to constraints

$$4X_1 - 6X_2 - 5X_3 + 4X_4 \geq -20$$
$$3X_1 - 2X_2 + 4X_3 + X_4 \leq 10$$
$$8X_1 - 3X_2 + 3X_3 + 2X_4 \leq 20$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Ans. Unbounded Solution

14. Maximize $Z = 3X_1 + 9X_2$ (Degeneracy)
Subject to constraints

$$X_1 + 4X_2 \leq 8$$
$$X_1 + 2X_2 \leq 4$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 0, X_2 = 2$, and Max $Z = 18$

15. A farmer has 1,000 acres of land on which he can grow corn, wheat or soyabeans. Each acre of corn costs Rs. 100 for preparation, requires 7 man-days of work and yields a profit of Rs. 30. An acre of wheat costs Rs. 120 to prepare, requires 10 man-days of work and yields a profit of Rs. 40. An acre of soyabeans costs Rs. 70 to prepare, requires 8 man-days of work and yields a profit of Rs. 20. If the farmer has Rs. 1,00,000 for preparation and can count on 8,000 man-days of work, how many acres should be allocated to each crop to maximize the profits ? Formulate this as a problem in linear programming and solve it by the simplex method.
Ans. X_1, X_2, X_3 = Acreage of corn, wheat and soyabeans, respectively,
Maximize(Total Profit) $Z = 30X_1 + 40X_2 + 20X_3$
Subject to constraints

$$10X_1 + 12X_2 + 7X_3 \leq 10{,}000$$
$$7X_1 + 10X_2 + 8X_3 \leq 8{,}000$$
$$X_1 + X_2 + X_3 \leq 1{,}000$$
$$X_1, X_2, X_3 \geq 0$$

X_1 = Acreage of corn = 250, X_2 = Acreage of wheat = 625, X_3 = Acreage of soyabeans = 0, and Maximize(Total Profit) Z = Rs. 32,500

Last date of submission to concern lab teacher is on or before : **4th August 2018**

ASSIGNMENT NO - 5
Tutorials based on LP Big M and Two Phase Method

Q.1. Explain the term "Artificial variable" and its use in linear programming.

Q.2. Why do we need Artificial variable ? Describe briefly the two-phase method of solving an LP problem with artificial variable.

Q.3. Big M method is also known as Penalty method. Justify.

Solve the following LP Problems by using Penalty (Big M) Method

1. Minimize $Z = 3X_1 - X_2$

Subject to constraints

$$2X_1 + X_2 \geq 2$$
$$X_1 + 3X_2 \leq 3$$
$$X_2 \geq 4$$
$$X_1, X_2, \geq 0$$

Ans. $X_1 = 3$, $X_2 = 0$, and Max $Z^* = -9$ or Mini $Z = 9$

2. Minimize $Z = 2X_1 + X_2$

Subject to constraints

$$3X_1 + X_2 = 3$$
$$4X_1 + 3X_2 \geq 6$$
$$X_1 + 2X_2 \leq 4$$
$$X_1, X_2$$

Ans. $X_1 = 3/5$, $X_2 = 6/5$, and Mini $Z = 12/5$

3. Minimize $Z = 3X_1 + 2X_2 + 3X_3$

Subject to constraints

$$2X_1 + X_2 + X_3 \leq 2$$
$$3X_1 + 4X_2 + 2X_3 \geq 8$$
$$X_1, X_2, X_3 \geq 0$$

Ans.

4. Maximize $Z = X_1 + X_2 + X_4$

Subject to constraints

$$X_1 + X_2 + X_3 + X_4 = 4$$
$$X_1 + 2X_2 + X_3 + X_4 = 4$$
$$X_1 + 2X_2 + X_3 = 4$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Ans. $X_1 = 4$, $X_2 = 0$, $X_3 = 0$, $X_4 = 0$, Max $Z = 4$

5. Minimize $Z = X_1 - 3X_2 + 2X_3$

Subject to constraints

$$3X_1 - X_2 + 2X_3 \leq 7$$
$$-2X_1 + 4X_2 \leq 12$$
$$-4X_1 + 3X_2 + 8X_3 \leq 10$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 4$, $X_2 = 5$, $X_3 = 0$ and Mini $Z = -11$

6. Maximize $Z = 2X_1 + X_2 + 3X_3$

Subject to constraints

$$X_1 + X_2 + 2X_3 \leq 5$$
$$2X_1 + 3X_2 + 4X_3 = 12$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 3$, $X_2 = 2$, $X_3 = 0$ and Max $Z = 8$

Solve the following LP Problems by using Two Phase Method

7. Minimize $Z = X_1 + X_2$

Subject to constraints

$$2X_1 + X_2 \geq 4$$
$$X_1 + 7X_2 \geq 7$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 21 / 13$, $X_2 = 10 / 13$, and Max $Z^* = -31 / 13$ or Mini $Z = 31 / 13$

8. Minimize $Z = X_1 - 2X_2 - 3X_3$

Subject to constraints

$$-2X_1 + X_2 + 2X_3 = 2$$
$$2X_1 + 3X_2 + 4X_3 = 1$$
$$X_1, X_2, X_3 \geq 0$$

Ans. Problem has no feasible solution. Refer Exa. 11 for answer.

9. Minimize $Z = (15/2)\, X_1 - 3X_2$

Subject to constraints

$$3X_1 - X_2 - X_3 \geq 3$$
$$X_1 - X_2 + X_3 \geq 2$$
$$X_1, X_2, X_3 \geq 0$$

Ans. $X_1 = 5/4$, $X_2 = 0$, $X_3 = 0$, and Mini $Z = 75/8$

10. Maximize $Z = 5X_1 + 8X_2$

Subject to constraints

$$3X_1 + 2X_2 \geq 3$$
$$X_1 + 4X_2 \geq 4$$
$$X_1 + X_2 \leq 5$$
$$X_1, X_2, \geq 0$$

Ans. $X_1 = 0$, $X_2 = 5$, and Max $Z = 40$

11. Minimize $Z = 3X_1 + 2X_2$

Subject to constraints

$$2X_1 + X_2 \geq 2$$
$$3X_1 + 4X_2 \geq 12$$
$$X_1, X_2 \geq 0$$

Ans. All $C_j - Z_j \geq 0$ but $Z = -4 < 0$ and artificial variable $A_1 = 4$ appears in the basis with positive value. Thus the given LP problem has no feasible solution.

12. Maximize $Z = 3X_1 - X_2$

Subject to constraints

$$2X_1 + X_2 \geq 2$$
$$X_1 + 3X_2 \leq 2$$
$$X_2 \leq 4$$
$$X_1, X_2 \geq 0$$

Ans. $X_1 = 1$, $X_2 = 0$, and Max $Z = 6$

Reference Books :

Operation Research By J K Sharma
Operation Research By V K Kapoor
Operation Research By Hamdy A. Taha

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ASSIGNMENT NO - 6
Tutorials based on LP Duality Method

Construct the dual of following primal LP Problems

1. Minimize $Z = 5X_1 - 6X_2 + 4X_3$

   Subject to constraints

   $$\begin{aligned} 3X_1 + 4X_2 + 6X_3 &\geq 9 \\ X_1 + 3X_2 + 2X_3 &\geq 5 \\ 7X_1 - 2X_2 - X_3 &\leq 10 \\ X_1 - 2X_2 + 4X_3 &\geq 4 \\ 2X_1 + 5X_2 - 3X_3 &= 3 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

2. Minimize $Z = 3X_1 - 2X_2 + 4X_3$

   Subject to constraints

   $$\begin{aligned} 3X_1 + 5X_2 + 4X_3 &\geq 7 \\ 6X_1 + X_2 + 3X_3 &\geq 4 \\ 7X_1 - 2X_2 - X_3 &\leq 10 \\ X_1 - 2X_2 + 5X_3 &\geq 3 \\ 4X_1 + 7X_2 - 2X_3 &\geq 2 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

3. Minimize $Z = X_1 - X_2 + 3X_3$

   Subject to constraints

   $$\begin{aligned} X_1 + X_2 + X_3 &\leq 10 \\ 2X_1 - X_3 &\leq 2 \\ 2X_1 - 2X_2 - 3X_3 &\leq 6 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

4. Minimize $Z = X_1 + X_2$

   Subject to constraints

   $$\begin{aligned} 2X_1 + 4X_2 &\leq 160 \\ X_1 - X_2 &= 30 \\ X_1 &\geq 10 \\ X_1, X_2 &\geq 0 \end{aligned}$$

5. Maximize $Z = 3X_1 + X_2 + 2X_3 - X_4$

   Subject to constraints

   $$\begin{aligned} 2X_1 - X_2 + 3X_3 + X_4 &= 1 \\ X_1 + X_2 - X_3 + X_4 &= 3 \end{aligned}$$

   $X_1, X_2, \geq 0$ and X_3, X_4 unrestricted in sign

6. Minimize $Z = X_1 - 3X_2 - 2X_3$

   Subject to constraints

   $$\begin{aligned} 3X_1 - X_2 + 2X_3 &\leq 7 \\ 2X_1 - 4X_2 &\geq 12 \\ -4X_1 + 3X_2 + 8X_3 &= 10 \end{aligned}$$

   $X_1, X_2, \geq 0$ and X_3 unrestricted in sign

7. A firm manufactures two types of products A and B on machine I and II as shown below :

Machine	Product A	Product B	Available Hours
I	30	20	300
II	5	10	110
Profit / Unit (Rs.)	6	8	

   The total time available is 300 hours and 110 hours on machines I and II respectively. Product A and B contribute Rs 6 and Rs 8 per unit respectively. Determine the optimum product mix. Write the dual of the problem and give its economic interpretation.

8. Food A contains 20 units of vitamin X and 40 units of vitamin Y per gram. Food B contains 30 units each of vitamin X and Y. The daily minimum human requirements of vitamin X and Y are 900 units and 1200 units respectively. How many grams of each type of food should be consumed so as to minimize the cost if food A costs Rs. 0.60 per gram and food B costs Rs 0.80 per gram ? Write the dual of the problem and give its economic interpretation.
9. What is duality ? State the advantages of Duality.
10. What is marginal value in duality ? State the rules for constructing the dual from primal.

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ASSIGNMENT NO - 7
Tutorials based on LP Sensitivity Analysis

1. A company wants to produce three products A, B and C. The unit profits on these products are Rs. 4 and Rs. 2 respectively. These products require two types of resources, manpower and raw material. The LP model formulated for determining the optimal product mix is as follows :

   Maximize $Z = 4X_1 + 6X_2 + 2X_3$

   Subject to constraints

   Man power constraint

   $$X_1 + X_2 + X_3 \leq 3$$

   Raw material constraint

   $$X_1 + 4X_2 + 7X_3 \leq 9$$
   $$X_1, X_2, X_3 \geq 0$$

   Where X_1, X_2, X_3 = Number of units of product A, B and C respectively to be produced.

(a) Find the optimal product mix and the corresponding profit to the company.

(b) Find the range on the profit contribution of product C (i.e. coefficient C_3 of variable X_3) in the objective function such that current optimal product mix remains unchanged.

(c) What shall be the new optimal product mix when profit per unit from product C is increased from Rs 2 to Rs 10 ?

(d) Find the range on the profit contribution of product A (i.e. coefficient C_1 of variable X_1) in the objective function such that current optimal product mix remains unchanged.

(e) What shall be the new optimal product mix when profit per unit from product A is increased from Rs 4 to Rs 9 ?

(f) Find the effect on optimality of the current solution when objective function is changed to Maximize $Z = 3X_1 + 8X_2 + 5X_3$

2. Solve the following LP Problems

   Maximize $Z = 4X_1 + 6X_2 + 2X_3$

   Subject to constraints

   $$X_1 + X_2 + X_3 \leq 3$$
   $$X_1 + 4X_2 + 7X_3 \leq 9 \text{ and } X_1, X_2, X_3 \geq 0$$

(a) Discuss the effect of change in the availability of resources from $[3, 9]^T$ to $[9, 6]^T$

(b) Which resource should be increased or decreased and how much to achieve less the best marginal increase in the value of the objective function ?

3. Solve the following LP Problems

   Maximize $Z = -X_1 + 3X_2 - 2X_3$

   Subject to constraints

   $$3X_1 - X_2 + 2X_3 \leq 7$$
   $$-2X_1 + 4X_2 \leq 12$$
   $$-4X_1 + 3X_2 + 8X_3 \leq 10 \text{ and } X_1, X_2, X_3 \geq 0$$

(a) Determine the separate range for discrete changes in a_{13} and a_{23} consistent with the optimal solution of the given LP Problem.

(b) a_1 (i.e. X_1 – column) is changed from $[3, -2, -4]^T$ to $[3, 2, -4]^T$

(c) a_3 (i.e. X_3 – column) is changed from $[2, 0, 8]^T$ to $[3, 1, 6]^T$

4. Discuss the effect on optimality by adding a new variable to the following LP Problem with column coefficients $[3, 3, 3]^T$ and coefficient in the objective function, 5

Minimize $Z = 3X_1 + 8X_2$

Subject to constraints

$X_1 + X_2 = 200$

$X_1 \leq 12$

$X_2 \geq 60$ and $X_1, X_2, \geq 0$

5. Solve the following LP Problems

Maximize $Z = 3X_1 + 5X_2$

Subject to constraints

$3X_1 + 2X_2 \leq 18$

$X_1 \leq 4$

$X_2 \leq 6$ and $X_1, X_2, \geq 0$

(a) If variable X_3 is added to the given LP problem, then obtain an optimal solution to the resulting LP Problem. It is given that the coefficients of X_3 in the constraint of the problem are 1, 1 and 1and its coefficient in the objective function is 2.

(b) Discuss the effect on the optimal basic feasible solution by adding a new constraint $2X_1 + X_2 \leq 8$ to the given set of constraints.

6. Optimum simplex table of the following linear programming problem has been given in the table.

Maximize $Z = 60X_1 + 20X_2 + 80X_3$

Subject to constraints

$6X_1 + 3X_2 + 5X_3 \leq 750$

$3X_1 + 4X_2 + 5X_3 \leq 600$

$X_1, X_2, X_3 \geq 0$

		C_j	60	20	80	0	0
C_B	Basic Variable B	Solution Value = X_B = B	X_1	X_2	X_3	S_1	S_2
60	X_1	50	1	− 1/3	0	1/3	− 1/3
80	X_2	90	0	1	1	− 1/5	2/5

(a) If the RHS of the constraints changes to $[750, 900]^T$, does it affect the optimum solution ? If yes,then find the optimum solution using sensitivity analysis approach.

(b) If coefficient of X2 in the constraints change to $[1, 1]^T$, does it affect the optimum solution? If yes, obtain the optimum solution using sensitivity analysis approach.

(c) If new constraint $X1+X2+X3 \leq 90$ is added to the LP Problem, does it affect the optimum solution ? If yes, obtain the optimum solution using sensitivity analysis approach.

7. The following tableau for a maximization type LPP is produced after few iterations of simplex method;

		8.5	10.5	0	0	0
Mix	Qty (bi)	X1	X2	S1	S2	S3
X2	300	0	1	3/5	-2/5	0
X1	300	1	0	-2/5	3/5	0
S3	400	0	0	-1/5	-1/5	1

Answer the following questions with brief reasons from above table;

(a) Does the tableau represent an optimal solution? If not, carry out necessary iterations and obtain an optimal solution.

(b) Is this solution degenerate?

(c) Are there more than single optimal solution to above problem ?

(d) What are the shadow prices or dual values of resources ?

(e) What is optimum objective function value for the problem ?

(f) If S1 represent the slack for production capacity constraint, how much should company be willing to pay for each additional unit of production capacity?

8. What do you understand by term "Sensitivity Analysis". Discuss the effect of

1) variation of C_j
2) variation of b_i
3) addition of a new constraint.

Reference Books :

Operation Research By J K Sharma
Operation Research By V K Kapoor
Operation Research By Hamdy A. Taha

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