Shantilal Shah Engineering College , Bhavnagar

Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (2110015)**

Tutorial : 1 **Topic : Matrices and Linear Equations (Ex-1 to 14)**

Ex-1 In each part determine whether the following matrix is in row echelon form, reduced row echelon form, both or neither :

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -6 & 4 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 7 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Ex-2 Find the a row echelon form (Gauss-elimination method) of the following Matrices & find it's rank also:

$$\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 1 & -1 & 2 \\ 1 & -1 & 1 & 2 \\ 2 & 2 & 1 & 6 \end{bmatrix}. \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Ex-3 Find the a reduced row echelon form (Gauss Jordan -elimination method) of the following Matrices& find it's rank also:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{bmatrix}, \begin{bmatrix} 3 & 1 & -1 & 2 \\ 1 & -1 & 1 & 2 \\ 2 & 2 & 1 & 6 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 1 & 0 & 2 \\ 2 & -2 & 0 & 2 & 2 \\ -1 & 1 & 2 & -3 & 1 \\ -2 & 2 & 1 & -3 & -1 \end{bmatrix}$$

Ex-4 Solve each of the following system by Gauss elimination method:

(i) $x+y+z=9, x+y-z=1, 2x+2y-2z=0.$

(ii) $3x+y-3z=13, 2x-3y+7z=5, 2x+19y-47z=32.$

(iii) $-x+3y+4z=30, 3x+2y-z=9, 2x-y+2z=10.$`

(iv) $-a+3b-4c=3, 3a+2b-c=8, 2a-b+2c=1.$

Ex-5 Investigate for what value of λ and η the equations

$x+2y+z=8, 2x+2y+2z=13, 3x+4y+\lambda z=\eta.$

Have (i) no solution (ii) unique Solution (iii) many solution.

Ex-6 Find the number of parameters in general solution of Ax=0 if A is 5x7 Matrix of rank 3.

EX-7 If A is an mxn matrix ,what is the largest possible value for it's rank ?.

Ex-8 Examine the consistency of the following system of equations & solve them, If they are consistent.

(i) $x+4y+4z=2, 2x+4y+z=1, x+5y+2z=-1, x+2y+8z=8.$

(ii) $x-2y+z-3w=-3, -3x+y-z+2w=2, 4x+3y-3z+w=1.$

(iii) $5x+3y+4z=0, 2x-y+z=0, 3x+y+2z=0.$

(iv) $3x+4y-z=5, x+y-z=4, 3x-4y+z=3.$

Ex-9 Solve the following system equation by Gauss Jordan-elimination method:

(i) $x+2y+z=3, 2x+y+3z=5, 2x+4y2z=7.$

(ii) $2x+y-z=2, x-3y+z=1.$

Ex-10 Find the inverse of following matrices by Gauss Jordan method:

$$(i)\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix}, (ii)\begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}. (iii)\begin{bmatrix} 3 & -1 & 5 \\ 2 & 6 & 4 \\ 5 & 5 & 9 \end{bmatrix} \text{(iv)} \begin{bmatrix} -1 & 2 & 3 \\ -4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix}$$

Ex-11 Find the rank of the matrix A in terms of determinant,where A= $\begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ -6 & 3 & -8 \end{bmatrix}$

Ex-12 Define the following:

Rank of matrix, Singular matrix ,Invertible matrix, Symmetric matrix.

EX-13 Use Cramer's rule to find the solution of the system

(1) $x+2y+z=3, 2x+y+3z=5, 2x+4y2z=7$

(2) $x-4y+z=6, 4x-y+2z=-1, 2x+2y-3z=-20$.

EX-14 Find the rank of the given matrix by reducing it to normal form.

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}.$$

Shantilal Shah Engineering College ,Bhavnagar

Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (2110015)**

Tutorial :2 **Topic : Vector in Rn ,V.Space, Subspace & basis (Ex-1 to 16)**

Ex-1 Find a vector in R^2 with Euclidean norm zero,and whose inner product with (-3,1) is zero.

Ex-2 State and prove Schwartz inequality and triangle inequality.
Verify it for the following vectors.
(1)$u=(0,-2,2,1)$ & $v=(-1,-1,1,1)$.
(2)$u=(0,-2,2)$ & $v=(-1,-1,1)$.
(3)$u=(0,2,2,1)$ & $v=(1,-1,1,1)$.
(4)$u=(0,-2,2,5)$ & $v=(-1,-1,1,0)$.
(5)$u=(0,-2,2,1,-2)$ & $v=(-1,-1,1,1,2)$.

Ex-3 State and prove Pythagorean theorem
show that $u=(3,0,1,0,4,-1)$ & $v=(-2,5,0,-3,-18)$ are orthogonal and verify Pythagorean theorem hold.

Ex-4 Find the value of k for $u=(k,k,1)$ & $v=(k,4,3)$ if u & v are orthogonal.

Ex-5 Prove that $uv=\frac{1}{4}\|u+v\|^2-\frac{1}{4}\|u-v\|^2$. where $u,v\in R^n$.

Ex-6 Show that the set of polynomials(degree at most n) with real coefficients is vector space

Ex-7 Show that set of all 2x2 matrices of the form $\begin{bmatrix} a & l \\ l & b \end{bmatrix}$ with

addition define by $\begin{bmatrix} a & l \\ l & b \end{bmatrix}+\begin{bmatrix} c & l \\ l & d \end{bmatrix}=\begin{bmatrix} a+c & l \\ l & b+d \end{bmatrix}$ and

scalar multiplication $k\begin{bmatrix} a & l \\ l & b \end{bmatrix}=\begin{bmatrix} ka & l \\ l & kb \end{bmatrix}$ is a vector space ,

where all the entries are from R.

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Ex-8 Check whether $V=R^n$ is a vector space with respect to the operation
$(u_1,u_2)+(v_1,v_2)=(u_1+v_1-2,u_2+v_2-3)$ &
$\alpha(u_1,u_2)=(\alpha u_1+2\alpha_1-2,\alpha u_2-3\alpha+3)$.

Ex-9 Show that $V\in R^n$ with the standard vector addition and scalar multiplication define as $\alpha(u_1,u_2)=(u_1,\alpha u_2)$ is not a vector space.

Ex-10 Show that W is a subspace of P_3 ,the set of all polynomials of degree 3 or less.

Ex-11 Which of the following are subspace of some Euclidian space.
$(a) S=\{(a+b,3a-b,2a+b)/a,b\in R\}$
$(b) S=\{(x,y)/x=3y\}$
$(c) S=\{(x,y,z)/x+y-z=0,4x-2y+z=2\}$
$(d) S=\{(x,y)/xy\geq 0\}$
$(e) S=\{(x,y,z)\in R^3/x^2+y^2+z^2=1\}$
$(f) S=\{A_{n\times n}/Ax=0\Rightarrow x=0\}$.
$(g) S=\{X\in M_{n\times n}/X^T=-X\}$
$(h) S=\{X\in M_{n\times n}/X^TX=I\}$

Ex-12.Determine the following polynomials span of P_2:
$p_1=1-x+2x^2,\ p_2=5-x+4x^2,\ p_3=-2-2x+2x^2$.

Ex-13 Determine whether $v_1=(1,1,2),v_2=(1,0,1)$ & $v_3=(2,1,3)$ Span the vector space R^3.

Ex-14 Show that the set $S=\{e^x,xe^x,x^2e^x\}$ in $C^2(-\infty,\infty)$ is linearly Independent.

EX-15 Show that $S=\left\{\begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix},\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix},\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}\right\}$ is a basis for M_{22}

Ex-16 Find the standard basis vector that can be added to the set $s=\{(1,0,3),(2,1,4)\}$ to produce a basis for R^3.

Ex-16 Show that the set $s=\{(1,2),(3,-1),(1,0)\}$ is not basis for R^2.

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Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (2110015)**

Tutorial :3 **Topic : Inner Product Spaces**

Ex-1 Define Inner Product Space and Determine which of the following are inner Product on R^3 if $u=(u_1, u_2, u_3)$, $v=(v_1,v_2, v_3)$,

(i) $< u,v >= 2u_1v_1 + u_2v_2 + 4u_3v_3$,

(ii) $< u,v > = u_1^2v_1^2 + u_2^2v_2^2 + u_3^2v_3^2$.

Ex-2 If $u = (u_1,u_2), v = (v_1,v_2)$ are vectors in R^2 then prove that R^2 is an inne product space with respect to the inner product defined as $< u,v > = 4u_1v_1 + u_2v_1 + 4u_1v_2 + 4u_2v_2$.

Ex-3 Show that $< u,v > = 9u_1v_1 + 4u_2v_2$ is the inner product on R^2 generated by the matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

Ex-4 Let $u = (u_1,u_2), v = (v_1,v_2)$. find a matrix that generates the Following inner products.

(i) $< u,v > = 3u_1v_1 + 5u_2v_2$, (ii) $< u,v > = 4u_1v_1 + 6u_2v_2$.

Ex-5 Let R^3 have the Euclidean inner product. Use Gram-Schmidt Process to transform the basis vectors

$u_1 = (1,0,0), u_2 = (3,7,-2), u_3 = (0,4,1)$

into an orthonormal basis.(IMP)

Ex-6 Use Gram-Schmidt process to transform the basis $\{1,x,x^2\}$ of P_2 into an orthonormal basis

If (a) $< p,q >= p(0)q(0) + p(1)q(1) + p(2)q(2)$,

(b) $< p,q >= \int_0^2 p(x)q(x)dx$.(IMP)

Ex-7 Find a basis for the orthogonal complement of the subspace W of the corresponding space R^n spanned by the vectors

(i) $u_1=(2,0,-1)$ (ii) $u_2=(4,0,-2)$ in R^3.

Ex-8 Use Gram-Schmidt process to transform the basis

$\left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ of M_{22}

into an orthogonal basis if $< A,B >= tr(AB^T)$.

Ex-9 Find the Least squares solution of the system Ax=b given by

$x_1 + x_2 = 7, -x_1 + x_2 = 0, -x_1 + 2x_2 = -7$.

Ex-10 Find the Least squares solution of the system Ax=B given by

$x_1 - x_2 = 4, 3x_1 + 2x_2 = 1, -2x_1 + 4x_2 = 3$.

Ex-11 Let the vector space P_2 have the inner product

$$\langle p,q \rangle = \int_{-1}^{1} p(x)q(x)dx$$

(i) Find $\|p\|$ for $p = x^2$,(ii) Find $d(p,q)$ if $p = 1 \& q = x$.

Ex-12 For $U = \begin{bmatrix} u_1 & u_2 \\ u_3 & u_4 \end{bmatrix} \& V = \begin{bmatrix} v_1 & v_2 \\ v_3 & v_4 \end{bmatrix}$ in M_{22} ,define

$\langle U,V \rangle = u_1v_1 + u_2v_2 + u_3v_3 + u_4v_4$ for the matrices $A \& B$,verify Cauchy-Schwartz inequality and find the cosine of the angle between them ,if $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix} \& B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$.

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Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (110015)**

Tutorial :4 **Topic : Linear Transformations(Ex-1 to 12)**

Ex-1 Determine whether the following functions are linear transformations:

(i) $T: M_{mn} \to M_{nm}$, where $T(A) = A^T$.

(ii) $T: M_{nn} \to R$, where $T(A) = \det(A)$.

(iii) $T: M_{nn} \to R$, where $T(A) = tr(A)$.

Ex-2 Consider the basis S={v_1,v_2}for R^2,where v_1=(1,1) and v_2=(1,0) and let $T: R^2 \to R^2$ be the linear transformation such that $T(v_1) = (1,-2)$ and $T(v_2) = (-4,1)$.
Find a formula for $T(x_1, x_2)$ and use the formula to find $T(5,-3)$.

Ex-3 Let $T_1: P_2 \to P_2$ and $T_2: P_2 \to P_2$ be the linear transformation given by $T_1\big(p(x)\big) = p(x+1)$and $T_2\big(p(x)\big) = xp(x)$.
Find $(T_2 o T_1)(a_0 + a_1 x + a_2 x^2)$.

Ex-4 Let $T: P_2 \to R^2$ be the linear transformation define by $T(a_0 + a_1 x + a_2 x^2)$ =$(a_0 - a_1, a_1 + a_2)$ the

(I) Find a basis for ker(T),

(II) Find a basis for R(T),

(III)Verify the dimension theorem.

Ex-5 In each case, Determine whether the linear transformation is one to one ,onto,or both, or neithr

.(I) $T: R^2 \to R^2$ where $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$.

(II) $T: R^3 \to R^3$ where $T(x_1, x_2, x_3) = (x_1 + 3x_2, x_2, x_3 + 2x_1)$.

Ex-6 Let $T: R^3 \to R^3$ be the linear operator define by the formula
$T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 2x_2 - x_3, 3x_2 + 2x_1)$,
Determine whether T is one to one. If So, find $T^{-1}(x_1, x_2, x_3)$.

Ex-7 Find the rank and nullity of transformation $T: R^3 \to R^2$,
$T(x,y,z) = (x+y+z, x+y)$.

Ex-8 Let $T: R^2 \to R^3$ be the linear transformation define by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$,
Find the matrix of the transformation T w.r.t. the bases S_1={ v_1,v_2} for R^2 and S_2={w_1,w_2,w_3} for R^3,
where $v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, w_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, w_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

Ex-9 Find the associate matrix of the linear transformation $T: R^3 \to R^3$,
$T(u_1, u_2, u_3) = (u_1 + u_2, u_2 + u_3, u_1 + u_3)$ with
$B = \{(1,0,0),(1,1,0),(1,1,1)\} \& B' = \{(1,0,1),(0,1,0),(1,0,-1)\}$
for the domain and co-domain of T respectively.

Ex-10 If $T: P_2 \to P_3$ be the linear transformation define by $T(p(x)) = xp(x+1)$,then find the matrix $[T]_{B',B}$
w.r.t bases $B = \{1, x, x^2\} \& B' = \{1, x, x^2, x^3\}$.

Ex-11 Let $T: R^2 \to R^3$ be the linear transformation define by $T(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$
,find the matrix for the transformation T w.r.t the bases $B = \{u_1, u_2\} \& B' = \{v_1, v_2, v_3\}$ for R^2 and R^3 respt. Where
$u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

Ex-12 Using induced matrix associated with each transformation determine the new point after applying the transformation to the given point,

(a) x=(1,-2,1) reflcted about xz-plane.

(b) x=(9,4,-2) projected on the x-axis.

(c) x=(1,-3,) roteted 30^0 in the counter clock wise direction.

Shantilal Shah Engineering College , Bhavnagar

Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (2110015)**

Tutorial :5 **Topic : Vector Calculus (Ex-1 to 18)**

Ex-1 Find the Gradient $f(x,y,z)=2z^3-3(x^2+y^2)z+\tan^{-1}(xz)$ at (1,1,1).

Ex-2 If $r=x\hat{i}+y\hat{j}+z\hat{k}$,evaluate

$(a)\nabla r^n,(b)\nabla\left|r^3\right|,(c)\nabla r,(d)\nabla(\ln r),(e)\nabla(\frac{1}{r}).$

Ex-3 Find the angle between the surfaces $x^2+y^2+z^2=9$,and $z=x^2+y^2-3$ at the Point (2,-1,2)

Ex-4 Find the derivative of $f(x,y)=x^2\sin 2y$ at the point $\left(1,\frac{\pi}{2}\right)$ in the direction of $v=3\hat{i}-4\hat{j}$.

Ex-5 Find the directional derivative of the function f(x + y) = ax + by ; a,b are constants,at the point (0 ,0) which makes an angle of 30^0 with positive x-axis.

Ex-6 The temperature at any point in space is given by T = xy + yz + zx. Determine the derivative of the vector $3\hat{i}-4\hat{k}$ at the point (1, 1, 1).

Ex-7 Find the potential function for the field $F=e^{y+2z}\left(\hat{i}+x\hat{j}+2x\hat{k}\right)$.

Ex-8 Prove that $r^n r$ is irrotational.

Ex-9 Find the directional derivative of div F at (2 ,2,1) in the direction of normal to the sphere $x^2+y^2+z^2=9$,where $F=x^2z\hat{i}+xy^2\hat{j}+yz^2\hat{k}$.

Ex-10 Prove that $\nabla^2 f(r)=f''(r)+(2/r)f'(r)$.

Ex-11 Find the parametric eqations of the line joining (-3,2) and (2,-1).

Ex-12 Integrate $f(x,y,z)=x+\sqrt{y}-z^2$ over the path $c=c_1\cup c_2$ from (0,0,0)to (1,1,1)with $c_1:r(t)=t\hat{i}+t^2\hat{j},c_2:r(t)=\hat{i}+\hat{j}+t\hat{k},0\le t\le 1.$

Ex-13 Show that $\overline{F}=(y^2-z^2+3yz-2x)i+(3xz+2xy)j+(3xy-2xz+2z)k$ is both solenoid and irrotational.(IMP)

Ex-14 Find the value of n for which the vector $r^n\overline{r}$ is solenoidal , where $\overline{r}=xi+yj+zk$.

Ex-15 Show that $\overline{E}=\frac{\overline{r}}{r^2}$ is irrotational.

Ex-16 State Green's Theorem.Verify it for $\oint_C [(x^2-2xy)dx+(x^2y+3)dy]$, where C is Boundary of the region bounded by the parabola $y=x^2$ and $y=x$.

Ex-17 Evaluate $\oint_C [(x^2-coshy)dx+(y+sinx)dy]$ byGreens theorem,where C is the rectangle with vertices (0,0) ,(π, o), ,(π, 1),(0,1).

Ex-18 State only Stokes and Gauss-Divergence theorem.

Shantilal Shah Engineering College ,Bhavnagar

Sem-II (All Branches) **Sub : Vector Calculus & Linear Algebra (2110015)**

Tutorial :6 **Topic : Eigen values & Eigen vectors (Ex-1 to 18)**

Ex-1 Find the Eigen value and Eigen Vector for the following matrices:

(1) $\begin{bmatrix}5 & 3\\1 & 3\end{bmatrix}$ (2) $\begin{bmatrix}3 & 2 & 3\\0 & 6 & 10\\0 & 0 & 2\end{bmatrix}$ (3) $\begin{bmatrix}1 & -6 & -4\\0 & 4 & 2\\0 & -6 & -3\end{bmatrix}$ (4) $\begin{bmatrix}-420 & \frac{1}{2} & 576\\0 & 0 & 0.6\\0 & 0 & \sqrt{3}\end{bmatrix}$

Ex-2 Show that if $0 < \theta < \pi$,then A=$\begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}$ has no real Eigen Value and Consequently no Eigen Vectors.

Ex-3 For the matrix A=$\begin{bmatrix}2 & 0\\3 & 3\end{bmatrix}$,Find the Eigen Values of

(i) A (ii) A' (iii) A^2 (iv) A^{-1} (v) adjA (vi) 3A (vii) A^3+A^2+A+2I.

Ex-4 Find the eigen values and bases for the eigenspaces of A^{25} and A + 2I , where A = $\begin{bmatrix}3 & 0\\8 & -1\end{bmatrix}$

Ex-5 Define Algebraic and Geometric Multiplicity.

Determine the Algebraic and Geometric Multiplicity Of

(1) $\begin{bmatrix}3 & 0\\8 & -1\end{bmatrix}$, (2) $\begin{bmatrix}-2 & 2 & -3\\2 & 1 & -6\\-1 & -2 & 0\end{bmatrix}$.

Ex-6 Find the eigenvalues and eigen vectors of the matrix A and discuss about its algebraic and geometric multiplicity,

where A = $\begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$

Ex-7 Show that characteristic equation of a 2 x 2 matrix A can be expressed as $\lambda^2 - tr(A)\lambda + \det(A) = 0$.

Ex-8 State Caley-Hamilton theorem.Verift it for the folloing matrices

(1) $\begin{bmatrix}3 & 4\\1 & 2\end{bmatrix}$ (2) $\begin{bmatrix}1 & 1 & 2\\3 & 1 & 1\\2 & 3 & 1\end{bmatrix}$.

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Ex-9 Using Caley-Hamilton theorem,find A^2 ,A^{-1} ,A^{-2},A^{-3} if A = $\begin{bmatrix}1 & 4\\2 & 3\end{bmatrix}$.

Ex-10 Using Caley-Hamilton theorem if A=$\begin{bmatrix}2 & 1 & 1\\0 & 1 & 0\\1 & 1 & 2\end{bmatrix}$,then prove that

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix}8 & 5 & 5\\0 & 3 & 0\\5 & 5 & 8\end{bmatrix}.$$

Ex-11 Find an Orthogonal matrix P that diagonalizes A=$\begin{bmatrix}1 & 0\\-1 & 2\end{bmatrix}$, and hence find A^{10} also find Eigen value of A^2.

Ex-12 Find a matrix P that diagonalizes A and determine $P^{-1}AP$,where A is

(1) $\begin{bmatrix}-1 & 4 & -2\\-3 & 4 & 0\\-3 & 1 & 3\end{bmatrix}$, (2) $\begin{bmatrix}2 & -1 & -1\\-1 & 2 & -1\\-1 & -1 & 2\end{bmatrix}$,

Ex-13 Diagonalized orthogonally the matrices (i) $\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ (ii) $\begin{bmatrix}2 & 0 & 1\\0 & 3 & 0\\1 & 0 & 2\end{bmatrix}$.

Ex-14 Show that A=$\begin{bmatrix}\cos\theta & -\sin\theta & 0\\\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix}$ is orthogonal.

Ex-15 Reduce the quadratic form

$Q(x,y,z) = 3x^2 + 4xy + 3z^2 + 8xz + 8yz$ into canonical form.

EX-16 Describe the conic whose equation is

$5x^2 + 4xy + 8y^2 - 36 = 0$.

Ex-17 Translate and rotate the coordinate axis,if necessary , to put the conic $9x^2 - 4xy + 6y^2 - 10x - 20y = 5$ in standard position.Find the equation of conic in equation of conic in the final coordinate system.

Ex-18 Find a change of variable that reduces the quadratic form $2x^2 + 2y^2 - 2xy$ to a sum of Squares and express the quadratic form in terms of new variables.