SHANTLAL SHAH ENGINEERING COLLEGE, BHAVNAGAR

B.E. Sem-IV (MECHANICAL)

Sub : Complex Variable & Numerical Methods(2141905)

Tutorial :1 **Topic :1- Complex number & Analytic function**

Ex-1 Find the value of Re(f(z)) and Im(f(z)) for the function $f(z)=\frac{1}{1-2z}$ at $7+2i$

Ex-2 Find the principal argument of $z=\frac{-2}{1+\sqrt{3}i}$.

Ex-3 Find and plots all roots of $(1)\sqrt[3]{8i}\,(2)(-1)^{1/4}$.

Ex-4 Find and plot the cube roots of $1+i$ & Fifth roots of unity.

Ex-5 Show that if C is any root of unity other than unity iy self,then $1+C+C^2+C^3+C^4+\ldots\ldots+C^{n-1}=0$.

Ex-6 Find the real and imaginary part of $(-1-i)^7+(-1+i)^7$.

Ex-7 Define Domain and region. Is the Set $|z-1+2i|\le 2$ domain?

Ex-8 Define connected set. Sketch $S=\{z\,/\,-1<\operatorname{Im}(z)<2\}$ .Is it connected?

Ex-9 Find the roots of the equation $z^2-(5+i)z+8+i=0$.

Ex-10 Prove that $\lim_{z\to i}\frac{i\bar{z}}{3}=\frac{i}{3}$. by definition.

Ex-11 Evaluate $\lim_{z\to i}\frac{z^2+1}{z-i}$.

Ex-12 Show that $f(z)=\begin{cases}\frac{\operatorname{Re}(z^2)}{|z|}, & z\ne 0\\ 0, & z=0.\end{cases}$ is continuous at $z=0$.

Ex-13 State and Prove Cauchy-Reimann equations.

Ex-14 Is the function $f(z)=\begin{cases}\frac{\operatorname{Re}(z)}{z}, & z\ne 0\\ 0, & z=0.\end{cases}$ continuous at $z=0$?

Ex-15 Show that the function $f(z)=|z|^2$ is continuous everywhere but nowhere differentiable except at origin .

Ex-16 Show that neither $f(z)=|z|$ nor $f(z)=\bar{z}$ is an analytic function.

Ex-17 Check whether the following functions are analytic or not any point: $(a)\ f(z)=e^{\bar{z}}\ (b)\ f(z)=2x+ixy^2$.

.Ex-18 Show that for the function : $f(z)=\begin{cases}\frac{\bar{z}^3}{|z|}, & z\ne 0,\\ 0, & z=0.\end{cases}$ the C-R equations are satisfied at $z=0$,but not analytic at $z=0$.

Ex-19 Define Harmonic function . Is the function $u=x\sin x\cosh y-y\cos x\sinh y$ harmonic ?

Ex-20 Let a function $f(x)$ be analytic in a domain D. Prove that $f(z)$ must be constant in D if $f(x)$ is real valued for all z and $\overline{f}(x)$ is an analytic in D.

Ex-21 Construct the analytic function $f(z)$ when $(1)u=\sin x\cosh y\,(2)u=y^3-3xy^2+3x+1$.

Ex-22 If $f(z)=u+iv$ and $u-v=e^x(\cos y-\sin y)$,find $f(z)$ in term of z.

Ex-23 Find $f(z)=u+iv$,whose imaginary part is $\log(x^2+y^2)+x-y$.

Ex-24 Determine a & b such that $u=ax^3+bxy$ is harmonic and find its harmonic conjugate.

Ex-25 Show that $u=-x^3+2x+3xy^2$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$

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Tutorial :2 Topic :6- Interpolation (Ex-1 to 14)

Ex-1 Compute $\cosh(0.56)$ from the following data and estimate error :

x	0.5	0.6	0.7	0.8
$\cosh x$	1.127626	1.185465	1.255169	1.337435

Ex-2 Find the value of $\sin 52^o$ from the following table:

θ^o	45	50	55	60
$\sin \theta^o$	0.7071	0.7660	0.8192	0.8660

Ex-3 The table below gives the values of function $y = \tan x$.

Obtain the value of $\tan(0.40)$

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Ex-4 Find the third divided difference with arguments 2,4,9,10 of the function $f(x) = x^3 - 2x$

Ex-5 Compute $f(9.2)$ from the following values using NDD formula: ☹

x	8.0	9.0	9.5	11.0
$f(x)$	2.079442	2.1974225	2.251292	2.397895

Ex-6 From the following table, find $f(x)$ using NDD formula☹

x	1	2	7	8
$f(x)$	1	5	5	4

Ex-7 Determine the interploting polynomial of degree three using Langranges interpolation for the table .

x	-1	0	1	3
$f(x)$	2	1	0	-1

Ex-8 Find the Langrange interpolation polynomials from the following data :

x	0	1	4	5
$f(x)$	1	3	24	39

Ex-9 Employ Stirling formula to compute $y(35)$ from the following table:

x	20	30	40	50
y	512	439	346	243

Ex-10 Obtain the value of $f(8)$ & $f(15)$ from the following table:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Ex-11 Express the function $\dfrac{3x^2 - 12x + 11}{(x-1)(x-2)(x-3)}$ as a sum of partial fraction, using Lagrange's formula.

Ex-12 Using Langrange formula to fit a polynomials to the data:

x	-1	0	2	3
$f(x)$	8	3	1	12

Ex-13The shear stress in kips,per square foot(ksf) for 5 specimens in a clay stratum are:

Depth m	1.9	3.1	4.2	5.1	5.8
Stress- ksf	0.3	0.6	0.4	0.9	0.7

Use NDD interpolating polynomials to compute the stress at 4.5 m depth.

Ex-14 Let $f(40) = 836,\ f(50) = 682,\ f(60) = 436,\ f(70) = 272.$ Use Stirling formula to find $f(55)$.

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Tutorial :3 **Topic-9,10, 11 : Roots of equations & Soln of ODE**

Ex-1 Find the positive root of $x = \cos x$ correct upto three decimal places by **bisection method**.

Ex-2 Perform the five iteration of the **bisection method** to obtain a root of the equation $f(x) = \cos x - xe^{x} = 0$.

Ex-3 Find the positive root of $x^3 - 4x - 9 = 0$ using the bisection method In four stages.

Ex-4 Find the negative root of $x^3 - 7x + 3 = 0$ correct upto three decimal places by bisection method.

Ex-5 Using **Regula –Falsi method** ,find thr real root of the equation correct upto three decimal places .

Ex-6 Find the real root of the following equations correct upto three decimal places **By Iteration method** :

$(1)\, x^3 + x - 1 = 0 \quad (2)\, \cos x + 1 = 3x. \quad (3)\, x^3 - \cos x = 0.$

Ex-7 Obtain the **Newton-Raphson formula** from Taylor's theorem.

Ex-8 Develope a reccurence formula for finding $\sqrt{N}$,using Newton Raphson method ,hence find $\sqrt{27}$ to three decimal places.

Ex-9 Find root of the following equations by Newton-Raphson method :

$(1)\, x - \cos x = 0, x > 0$ to three decimal places.

$(2)\, \sin x = e^{-x}$ with $x_0 = 0.6$ to four decimal places.

Ex-10 Find a root of $x^4 - x^3 + 10x + 7 = 0$ correct to three decimal places between $a = -2$ & $b = -1$ by newton Raphson method.

Ex-11 Compute the real root of

(1) $f(x) = x - 2\sin x = 0$, starting from $x_0 = 2, x_1 = 1.9.$

(2) Cube root of 50

(3) $f(x) = x^3 - 2x - 1 = 0, x_0 = 1.5$ & $x_1 = 2.$ by the **secant method.**

Ex-12 Derive **secant method** and solve $xe^{x} - 1 = 0$ correct to three decimal Places between 0 and 1.

Ex-13 Using **Euler's method** to find: (1) $y(1.4)\, given\, \frac{dy}{dx} = xy^{1/2}, y(1) = 1$

(2) $y(0.2)\, given\, \frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$

(3) $y(1)\, given\, \frac{dy}{dx} = x + y, y(0) = 1.$

Ex-14 Using **Modified Euler's (Heun's) method** to solve $\frac{dy}{dx} = 1 - y, y(0) = 0$, And tabulate the solutions at $x = 0.1$ & 0.2. Compare the answer with Exact solution.

Ex-15 **Use second order Runge-Kutta method** to find:

$y(0.2)\, given\, \frac{dy}{dx} = x - y^2, y(0) = 1$ & $h = 0.1.$

Ex-16 Use **Fourth order Runge- Kutta method** to find:

(1) $y(0.2)\, given\, \frac{dy}{dx} = x + y, y(0) = 1$ & $h = 0.1.$

(2) $y(1.1)\, given\, \frac{dy}{dx} = x - y, y(1) = 1$ & $h = 0.05.$

(3) $y(0.2)\, given\, \frac{dy}{dx} = x^2 + y^2, y(0) = 1$ & $h = 0.1.$

17 Use **power method** to find the largest of Eigen values of the following matrix

(1) $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ (2) $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$, perform four iterations only.

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Tutorial :4 **Topic-2 &5 Complex Integration & Conformal mapping.**

Ex-1 Find an upper bound for the absolute value of the integral $\int_C e^z\,dz$, Where C is the line segment joining the points (0,0) and $(1,2\sqrt{2})$.

Ex-2 Evaluate $\int_C Re(z^2)dz$,where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clockwise direction.

Ex-3 Evaluate $\int_C f(z)dz$,where $f(z)$ os defined by $f(z) = \begin{cases} 1, if\ y < 0 \\ 4y, if\ y > 0, \end{cases}$ and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

Ex-4 Define Simply and multiply connected domains:

and State only (i) Cauchy's integral Theorem,(ii) Cauchy-Goursat's Theorem.

Ex-5 Using Cauchy's integral Theorem to Evaluate

$(a) \oint_C e^z\,dz, (b) \oint_C z^2 dz, n = 0,1,2,.., (c) \oint_C cosz\,dz.$

Ex-6 Evaluate: $\int_C \frac{1}{z^2+1}dz$, where C is $|z + i| = 1$,counterclockwise.

Ex-7 State Cauchy's integral formula: Using it Evaluate the following integrals,

$(a) \int_C \frac{e^z}{z+1}dz$, where C is circle $|z| = 2$.

$(b) \int_C \frac{e^z}{z(z-1)}dz$, where C is circle $|z| = 2$.

$(c) \int_C \frac{e^z}{(z-1)(z-3)}dz$, where C is circle $|z| = 2$.

Ex-8 Evaluate (1) $\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz\ where\ C : |z| = 3.$

(2) $\oint_c \frac{\sin 3z}{(z - \frac{\pi}{2})} dz\ where\ C : |z| = 5.$ by Cauchy's integral formula.

Ex-9 Evaluate :

$\oint_c \frac{e^z}{z(1-z)^3} dz. where\ C\ (1) |z| = 2, (2) |z| = \frac{1}{2}, (3) |z-1| = \frac{1}{2}.$

Ex-10 Find and sketch the image of region $x \geq 1$ under the transformation $w = \frac{1}{z}$.

Ex-11 Find the image of semi-infinite strip $x > 0, 0 < y < 2$,when $w = iz + 1$.Sketch The strip and its image.

Ex-12 Determine the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ Onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.

Ex-13 Find the image of the infinite strips

$(a) \frac{1}{4} \leq y \leq \frac{1}{2}$

$(b) 0 \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}.$

Ex-14 Define Mobius transformation.Find the image of the circle $|z| = 1$ in the w-plane under the Mobius transformation $w = \frac{z-i}{1-iz}$.

Also find the fix point of f.

Ex-15 Determine the bilinear transformation which mapping the points $0, \infty, i$ Into $\infty, 1, 0$.

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Tutorial :5 **Topic-3 & 4 : Power Sries and Application of Contour integration**

Ex-1 State Taylor's series and Laurent's series in complex plane:

Find Taylor's series and Laurent's series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0.

Ex-2 Find the Maclaurin's expansion of $f(z) = tanz$.

Ex-3 Determine the Laurent's series expansions of $f(z) = \frac{1}{(z+1)(z+3)}$

Valid for (1) $1 < |z| < 3$,(2) $|z| > 3$,(3) $0 < |z+1| < 2$, (4) $|z| < 1$.

Ex-4 Expand $\frac{1-e^z}{z}$ in Laurent 's series about z=0 and identify the singularities.

Ex-5 Define the following terms (a) Singular point (b) Regular point (c) pole

(d) Removable Singularity , (e) Essential Singularity (f) Zeroes (g) Residue.

Ex-6 Determine and classify all singularities of the following functions:

$(A) \quad f(z) = \frac{1}{z-z^3}$, (B) $f(z) = \frac{e^z}{z^2+1}$, (C) $f(z) = \frac{1-cosz}{z}$.

Ex- 7 Derive formula for Residue at simple pole.

Ex-8 Determine the residue at the pole for the following functions:

(I) $\frac{1}{(z+1)(z+3)}$, (II) $\frac{z+1}{(z^2-16)(z+3)}$, (III) $\frac{1-e^z}{z^4}$.(IV) $\frac{1-e^z}{z^3}$ $at z = 0$.

Ex- 9 Define residue at simple pole and find the sum of residues of the function

$f(z) = \frac{\sin z}{z\cos z}$ at its simple poles inside the circle $|z| = 2$.

Ex-10 State **Cauchy's Residue Theorem**: Using it Evaluate the following integrals.

(a) $\oint_C \frac{z^2 \, sinz}{4z^2-1}\, dz$, where C: $|z| = 2$.(b) $\oint_C \frac{cos\pi\, z^2}{(z-1)(z-2)}\, dz$, where C: $|z| = 3$.

(c) $\oint_C \frac{z^2}{(z-1)^2(z+2)}\, dz$, where C: $|z| = 3$.

Ex-11 Evaluate the following integrals by Residue theorem:

(1) $\oint_c \frac{dz}{\sinh 2z}, c : |z| = 2$ (2) $\oint_c \frac{5z-2}{z(z-1)} dz, c : |z| = 2$ (3) $\oint_c \frac{z^2 \sin z}{4z^2 - 1} dz, c : |z| = 2$.

Ex-12 Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue

At each pole. Hence Evaluate $\oint_c f(z) dz where, c : |z| = 3$.

Ex-13 Use Residue theorem to evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+1)(x^2+4)}$.

Ex-14 Evaluate : $\oint_c \frac{dz}{(z^2+1)^2}, c : |z+i| = 1$.

Ex-15Using the Residue Theorem, to Evaluate:

(1) $\int_0^{2\pi} \frac{1}{5-3sin\theta}\, d\theta$ (2) $\int_0^{2\pi} \frac{4\, d\theta}{5 + 4\sin\theta}$

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Tutorial :6 **Topic-7 &8 Numerical Int & Linear Algebric Equations**

Ex-1 State Trapezoidal rile with n=10 and evaluate(1) $\int_0^1 e^{-x^2}dx$. (2) $\int_0^1 e^{x}dx$.

Ex-2 Evaluate $\int_0^6 \frac{1}{1+x}dx$. taking h=1 using Simpson's $\frac{1}{3}$ rule .Hence obtain an approximate value of $\log_e 7$.

Ex-3 Evaluate $\int_0^3 \frac{1}{1+x}dx$. with n=6 using Simpson's $\frac{3}{8}$ rule and hence Calculate $\log_e 2$ .Estimate the bound of the error involve in the process.

Ex-4 The speed v meters per second ,of a car ,t seconds after it starts,is shown in the following table

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.60	21.60	18.54	10.26	4.50	4.5	5.4	9.0

Using Simpson's $\frac{1}{3}$ rule,find the distance travelled by the car in 2 minutes.

Ex-5 A river is 80 m wide .the depth d in meters at a distance x meters from one bank is given by following table .Calculate the are of cross-section of the river using Simpson's $\frac{1}{3}$ rule.

x	0	10	120	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Ex-6 Evaluate the integral $\int_4^{5.2} \log_e x dx$ using Simpson's $\frac{3}{8}$ rule.

Ex-7 Consider following tabular values

X	25.0	25.1	25.2	25.3	25.4	25.5	25.6
F(x)	3.205	3.217	3.232	3.245	3.268	3.268	3.280

Determine the area bounded by the given curve and X-axis between X=25 to X=25.6 by Trapezoidal rule and Weddle's rule.

Ex-8 Consider the following values,

X	10	11	12	13	14	15	16
y	1.02	0.94	0.89	0.79	0.71	0.62	0.55

Find $\int_{10}^{16} y dx$ by using Simpson's $\frac{1}{3}$ rule and Weddle's rule.

Ex-9 Evaluate:(1) $\int_1^3 \sin x dx$ using Gauss Quadrature of five points.

(2) $\int_0^1 e^{-x^2}dx$ by using Gauss Integration formula with n=3.

Ex-10 Solve the following equations using partial pivoting by Gauss-Elimination method.

(1) $x+y+z=9,\ 2x-3y+4z=13,\ 3x+4y+5z=0.$ (2) $-a+3b-4c=3,\ 3a+2b-c=8,\ 2a-b+2c=1.$

Ex-11 Solve the following equations by Gauss-Seidel method.

(1) $27x+6y-z=85,\ 6x+5y+2z=72,\ x+y+54z=110.$ (2) $10x+y+z=12,\ 2x+10y+z=13,\ 2x+2y+10z=14.$

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If the people do not believe that mathematics is simple ,it is only because They do not realise how complicated life is.